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Optical Measurements of Drops in Flows

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ABSTRACT

This article reviews recent developments of three selected techniques for the measurement of drops in fluid flows. In all cases the developments extend the capabilities and achievable measurement accuracy of the respective technique.

The first technique is the two-camera Depth from Defocus (DFD), in which drops within a volume are imaged onto two cameras with differing degrees of out-of-focus. This allows both drop size and position to be determined, yielding drop size distributions and volume concentration. The notable feature is the very well-defined detection volume, leading to high accuracy of the volume concentration measurement, a quantity which is very elusive with other measurement techniques.

The second technique is also an imaging technique in which the volume fraction of two-component drops of immiscible fluids can be determined. This technique is novel and has applications where drops and/or sprays of one fluid impact onto a film of another fluid and the mixture fraction of the resulting splashed, secondary droplets is sought.

The third technique is an extension of rainbow refractometry to a planar configuration, allowing drop size and temperature (refractive index) to be determined over an illuminated plane. Of particular interest is a novel approach to calibrating the scattering angle over the entire plane, an important input parameter for the inversion of the rainbow pattern into size and refractive index.

Keywords: sprays, atomization, drop imaging, rainbow refractometry, depth from defocus

1. DEPTH FROM DEFOCUS

There exist numerous implementations of depth from defocus imaging systems, starting from the introduction of the concept by Pentland [1] and Kratkov [2]. Some are based on single-sensor imaging [3, 4] or two-sensor imaging [5, 6]. A variation on DFD for particle tracking is astigmatism particle image velocimetry, using a cylindrical lens to distinguish whether the object is behind or in front of the object plane [7]. A more comprehensive summary of DFD techniques can be found in Zhou et al. [8]. In the present study two cameras are used, each imaging the same volume but with different degrees of out-of-focus. The principle of DFD is illustrated in Fig. 1.

Each drop/particle is therefore imaged out of focus



Figure 1. Principle of Depth from Defocus. a) Cameras are adjusted to be in focus at planes removed z_1 (yellow) and z_2 (green) from the object plane; b) Depending on drop position and camera, the image can be in or out of focus

and the aim of the processing is to mathematically describe the out of focus image such that the position and size of the drop can be extracted from the two images. The out of focus image is blurred and this image (I_i) can be described by the convolution of the

in-focus image (I_S) with a blur kernel h_i , i.e.

 $I_i = I_S * h_i$ (*i* = 1, 2 for cameras 1 and 2) (1)

The blur kernel is typically assumed to be Gaussian in nature with a standard deviaton σ , i.e.

$$h = \frac{1}{2\pi\sigma^2} \exp^{\frac{-r_h^2}{2\pi^2}} = \frac{1}{2\pi\sigma^2} \exp^{\frac{-r^2 + r_f^2 - 2rr_f \cos\theta}{2\pi^2}}$$
(2)

where r_h is a radial coordinate, as defined in Fig. 2 This allows the gray level g_t in the blurred imaged



Figure 2. Schematic of the blur convolution in the image. *M* is the magnification; d_p is the true particle/drop size; d_t is the particle image size using the gray threshold level g_t .

to be computed as a function of the true drop size d_0 , and rearranging into dimensionless quantities this takes the form

$$g_t = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\left(\frac{d_0}{2\sigma}\right)} \exp^{\frac{-\left[\left(\frac{r}{\sigma}\right)^2 + \left(\frac{rd_0}{d_0\sigma}\right)^2 - 2\frac{rd_0r_t}{\sigma\sigma d_0}\cos\theta\right]}{2}} d\left(\frac{r}{\sigma}\right)^2 d\theta$$
(3)

By involving a thresholding process on the blurred image with a fixed threshold g_t , a circular image with the measured diameter d_t can be obtained. Although the resulting integral of Eq. 3 cannot be calculated explicitly, it indicates a fixed functional relationship f between the dimensionless ratios $d_{t,i}/d_p$ and $(z - z_i)/d_p$ for a given threshold gray level g_t , where d_p is the true particle/drop diameter and $d_{t,i}$ is the directly measured diameter on the image, using the g_t gray level.

With two cameras, this leads to a classical problem of two unknowns $(d_p \text{ and } z)$ and two equations, namely:

$$\frac{d_{t,i}}{d_p} = f_i \left(\frac{z - z_i}{d_p} \right) \qquad (i = 1, 2) \tag{4}$$

The functions f_i are determined using a calibration reticule plate traversed through different z values. The bounds of the measurement depth range for each camera ($z_{i,min}$, $z_{i,max}$) correspond to the limit when $d_{t,i}$ goes to zero, i.e. when the particle can no longer be detected with a certain grey level threshold g_t on the image. The magnification must also be chosen

such that the expanded and blurred images of individual drops do not overlap with one another. The processing is depicted graphically in Fig. 3. A comprehensive study investigating the sensitivity of various optical parameters on the accuracy of the position and size determination of the drop can be found in [9]. Experience has shown that a normalized threshold gray level of 0.5-0.6 is a very good choice. One of the unique features of this imaging technique is the almost linear dependence of depth range with particle size, i.e. larger particles can be detected on the image for a larger variation of position z, or $\Delta w = z - z_i$. For instance, Fig. 4 shows a scatter diagram of approx. 20000 drops extracted from 2005 image pairs taken from a flat fan spray. The linear relationship between drop diameter and depth position (Δw) is evident from this diagram. The measurement volume is a function of particle size, but is approximately 4.5 x 3.5 x 25 mm³ (Width x Height x Depth) for the largest drops observed (240 μ m diameter). The backlight LED was set to pulse mode with a pulse width of 1 μ s.

Knowing the linear dependence of measurement depth on drop size, allows computation of the number density (number of drops per m³). Practically the volumetric number density is computed by first dividing the entire size range into sub-ranges, for example $N_{bin} = 100$ equal bins. There are n_i drops in the *i*th bin. The width (*W*) and height (*H*) are estimated as the image area minus the area containing truncated drop images, which cannot be processed, i.e. $W_i = W - d_i$, $H_i = H - d_i$, using the values of *W* and *H* for the camera with the smallest field of view. The depth of the measurement volume is equal to the effective depth of field for size $\overline{d_i}$, i.e. Δw_i . The number density \overline{N} can then be expressed as

$$\overline{N} = \frac{1}{N_{\text{imag}}} \sum_{i=1}^{N_{\text{bin}}} \frac{n_i}{W_i H_i \Delta w_i}$$
(5)

where N_{imag} is the number of processed images. The number density computed for the above-mentioned measurement of a spray from a flat fan nozzle is 357 drop/cm³. Further examples of laboratory measurements with this depth from defocus system can be found in the articles [8, 9].

2. VOLUME FRACTION OF TWO-COMPONENT DROPLETS

To illustrate the purpose of this technique, Fig. 5 illustrates the splash resulting when a red-colored water drop impacts onto a clear film of silicone oil of 600μ m thickness. The ejected secondary drops often have a mixture of water and silicone oil and the aim is to determine the volume fraction of the inner drops compared to the bulk drops. Such a measurement capability is interesting (and necessary) when examining the impact of fuel drops onto an oil film [10] in a combustion chamber or a water-urea drop onto an enriched urea film on the wall of a SCR sys-



Figure 3. Two images of the same particle/drop and the measured diameters on the images, d_t . The calibration functions f_1 and f_2 are shown on the right diagram, from which the size and position of the particle can be determined by solving Eqs. 4.



Figure 4. Sampled drop diameters and corresponding depths

tem (Selective Catalytic Reduction). Thus, this technique applies to drop impact onto films of dissimilar liquids [11, 12].

The technique determines the volume fraction of the



Figure 5. Photograph of a red-colored water drop (D=3mm, u=3.3 m/s) impinging onto a film of silicone oil of 600 μ m thickness, illustrating the generated secondary droplets; b),c) Time sequence of secondary droplets ejected from finger jets.

inner droplet to the outer drop $(V_{\text{frac}} = V_{\text{inner}}/V_{\text{outer}})$ by examining the area ratio on the image of red area to total area of each drop ($A_{\text{frac}} = A_{\text{inner}}/A_{\text{outer}}$). This however cannot be a direct correspondence, since the (red) area of the inner drop depends on the observation perspective. For instance, Fig. 6 illustrates images of such a two-component drop rotating freely in an acoustic levitator at different time instances. The area ratio A_{frac} changes dramatically, although the volume fraction V_{frac} remains constant. To understand this effect, a simple ray-tracing approach can be considered, illustrated in Fig. 7. It is obvious from these simple considerations, that the area ratio on a planar image will also depend on the relative refractive index of the outer drop to the surrounding medium, usually gas.

The application of this simple ray-tracing exercise



Figure 6. Levitated two-component droplet $(V_{\text{frac}}=0.13)$ of red colored water and silicone oil at various time instances.

leads to a relation between A_{frac} and V_{frac} as illustrated in Fig. 8, indicating that, depending on the position of the inner droplet in the outer drop with respect to the viewing angle, the maximum discernible volume fraction is a little as 0.08 when the inner drop



Figure 7. a) Schematic representation of ray paths refracted in a two-component droplet in a sectional view; b) projected image onto a camera; c) definition of coordinate system origin; d) definition of projected eccentricity vector.

is at the rear of the outer drop, and 0.58 when the inner droplet is at the front of the outer drop. This maximum is reached when the area ratio becomes unity. To alleviate this limitation, it is therefore necessary either to observed the droplet for a longer period of time, during which the observation perspective may change and the area ratio may sometimes be smaller than unity, or to observe the droplet simultaneously from more than one perspective. The latter approach has been used in the present study, realized in a onecamera configuration, as shown in Fig. 9. In this configuration, two orthogonal views of the drop are imaged simultaneously onto different portions of the camera chip using mirrors and a prism in front of the camera.

It becomes apparent that due to the influence of re-



Figure 8. a) Relation between V_{frac} and A_{frac} in dependence of the relative position of the inner droplet. Dashed lines represent the V_{frac} for which $A_{\text{frac}} = 1$; b) Light ray paths within a twocomponent droplet in sectional view, resulting in varying A_{frac} .

fraction inside the two-component droplet the real position of the enclosed droplet cannot be determined unambiguously. Only the projected eccentri-



Figure 9. Sketch of the splashing setup showing the respective fields of view (FOV), focal planes (FP) and depth of field (DOF).

cities $\vec{e}_{i,proj}$ can be measured directly from the recordings. This makes direct solution of the inverse problem difficult. On the other hand, the projected A_{ratio} in combination with $\vec{e}_{i,proj}$ do include information about V_{frac} . In order to utilize this information to determine the V_{frac} a support vector machine (SVM) is used. The SVM is a methodology from the field of machine learning, first introduced by [13], which has become widely used for solving classification problems [14]. The aim of using a SVM in the context of this study is to provide an algorithm, which is able to predict a volume fraction \hat{V}_{frac} based on experimentally observable features, summarized in an observation vector \vec{b} .

To train a SVM with the aim to predict \hat{V}_{frac} , first a set of classes has to be defined. It becomes apparent from Fig. 8 that the projected A_{ratio} is close to unity for high V_{frac} even if the inner droplet is positioned on the side facing the observer. The V_{frac} of cases with $A_{\text{ratio}} \approx 1$ cannot be determined unambiguously, since the projected image of the droplet contains limited information. A reasonable trade off is to limit the measuring range of V_{frac} between 0 and 0.5. This measuring range is then divided into equal bin sizes of 0.025 resulting in 21 discrete classes each representing 5% intervals of the upper measuring range limit.

The projected A_{ratio} depends on V_{frac} , the refractive index *n*, the relative position e_i of the inner drop and the aspect ratio ε of the two-component drop. With the aim to determine V_{frac} , the information of A_{ratio} , *n*, ε and e_i must therefore be taken into account. Since it is not possible to directly determine the exact position of the inner droplet from the recordings, its position is estimated by considering the centre of the projected area of the inner droplet in each perspective image $e_{i,proj}$, with $e_{i,proj}$ being a two-dimensional vector as depicted in Fig. 7d. Thus the observation vector \vec{b} becomes

$$\vec{b} = [\varepsilon, A_{\text{ratio},1}, A_{\text{ratio},2}, (6)$$

$$e_{x,\text{proj},1}, e_{z,\text{proj},1}, e_{y,\text{proj},2}, e_{z,\text{proj},2}].$$

The area ratios can be summarized in $A_{\text{ratio, p}}$ and the eccentricities in $e_{i,\text{proj},p}$, where the subscript $p \in [1, 2]$ denotes the respective perspective 1 or 2 and the subscript *i* denotes the components of the eccentricity vector. The classification is based on these seven features.

In the next step the algorithm needs to be trained, i.e. the hyperplanes for the l= 210 binary classifiers need to be found. For this purpose 67500 synthetic observations \vec{b} were generated. For each observation two orthogonal projections are generated. The V_{frac} and the position of the inner droplet e_i are randomly varied, whereby V_{frac} is limited to the interval [0, 0.5] and e_i is constrained by the condition that the inner droplet must be wholly within the outer droplet. An image processing script based on the Matlab image processing toolbox is then used to extract $A_{ratio,p}$, ε and $e_{i,proj,p}$ from the synthetically generated image pairs. These quantities are then combined in \vec{b} and each observation is labeled with the V_{frac} it was generated with. Further details of the SVM implementation can be found in [15].

This section closes with sample measurements to illustrate the achievable accuracy. These measurements were conducted on drops held in an acoustic levitator. Two types of experiments were conducted. In the first experiments, solid particles of known and precise size were injected into the outer drop; hence, the exact volume fraction is known once the volume of the outer drop is determined through shadowgraphy. This offers a ground truth as a benchmark and moreover, the solid particles cannot deform under influence of the acoustic pressure. The second experiments was then a drop-in-drop experiments, in which the volume of both the inner (colored) and outer drop was carefully controlled during injection with precision needles. Again, the volume fraction is therefore known. These experiments allow the accuracy of the technique under well-controlled conditions to be explored.

The results of these two experiments are shown in Fig. 10. These results indicate, that particularly for volume fractions < 0.3, the measurement accuracy is very good. For larger values of volume fraction the measurements performed with particles embedded in the outer drop are significantly better that for the drop-in-drop experiments. This is likely due to the deformation of the inner drop due to acoustic pressure, an effect which was not considered in the SVM training data.

3. PLANAR RAINBOW REFRACTO-METRY

Roth et al. [16] first proposed standard rainbow refractometry (SRR) to simultaneously measure the



Figure 10. Comparison between experimental and classification based volume fraction V_{frac} determination. Blue markers indicate drop-in-drop experiments with red colored water inside silicone oil, whereas red markers indicate spherical particles (D=1mm) inside silicone oil drops. The black symbols correspond to the estimated volume fraction from the experimental images. ID (Identification number) is simply a counter for the various experiments, arranged sequentially in order of increasing V_{frac} .

temperature (refractive index) and size of individual droplet (or monodisperse droplets), however, SRR is very sensitive to any departure from sphericity. With a similar optical system, van Beeck et al. [17] developed global rainbow refractometry (GRR) for the measurement of size distribution and average refractive index (or temperature) of an ensemble of spray droplets; GRR is relatively insensitive to nonsphericity droplets. Further to these conventional and partially commercially available techniques, the capability of rainbow refractometry to measure the following quantities has been demonstrated: size of droplets in air [18] and in liquid-liquid flows [19], non-sphericity [20], heat & mass transfer parameters (refractive index [21], temperature [22], transient evaporation rate [23]), often simultaneously. Recently, the technique has been extended to characterize oscillating [24] and colloidal droplets [25, 26]. In the following, some recent developments to extend rainbow refractometry from a point measurement to a planar measurement will be discussed, in particular with reference to advances outlined in [27, 28].

The optical configuration for a planar rainbow refractometer is shown in Fig. 11 and the experimental setup is pictured in Fig. 12. The illumination is with a vertically polarized laser light sheet, similar to that used in particle image velocimetry (PIV). The detector must now be aligned at an angle, such that the primary rainbow pattern for the particular relative refractive index of the drops will be captured on the camera chip. There are several complicating factors in doing this and resolving these factors is the substance of the present contribution.

To begin, the angular position a particular pixel on the camera chip represents depends on the position of the scattering drop in the illumination Therefore, the angle calibration must be plane. carried out throughout the entire plane. This will be accomplished using a chain of monodispersed droplets, as described below. Second, for several reasons the relative refractive index of the drop may not be known. This would be the case if there were a mixture of different fluids, and then the refractive index must be determined simultaneously with the size and position of the drop. Another situation would be if the drops were two or multi-component and one component was more volatile, such that evaporation would lead to a change of relative refractive index. The objective in analysing the rainbow pattern arising from each individual drop is therefore to determine the size, refractive index (temperature) and position in each recorded image. This analysis is often referred to as the 'inversion' of the rainbow pattern, since it represents an inverse problem to be solved.



Figure 11. Optical configuration for planar rainbow refractometry

An important element in the optical configuration is the insertion of a horizontal slit at the back focal plane of the focusing lens, in front of the camera. With the horizontal slit, rainbow patterns of drops at different heights and horizontal positions in the illuminated measurement plane are separately recorded on different row and column pixels of the camera. A typical image obtained from a spray using the planar rainbow refractometer is pictured in Fig. 13. From this image it is apparent, that the planar rainbow re-



Figure 12. Experimental setup to demonstrate planar rainbow refractometry

fractometry will be limited in measurable number density, since at higher concentration the rainbow patters will overlap excessively and can not be analysed. This is similar to limitations in interferometric particle imaging (IPI) [29]. Furthermore, it is also apparent that determination of the position of the particle is not straightforward, since the rainbow pattern is by nature more intense for the primary fringe (Airy) and less for the supernumerary fringes. More details regarding this difficulty can be found in [27]. A novel development in implementing the planar



Figure 13. Typical image obtained from a spray using the planar rainbow refractometer.

rainbow refractometer is the method of calibrating the scatter angle. For this a monodisperse droplet generator is used (vibrating orifice) [30] and traversed through a number of positions in the illumination plane, as shown in Fig. 14a. The resulting rainbow patters received for three different positions are pictured in Fig. 14b, directly next to one another. This image shows the rainbow pattern with supernumerary bows and also the ripple structure for the droplet stream, superimposed on the rainbow patterns for clarity. Knowing the position and size of the drops, it is then possible to compute the expected rainbow pattern using a Lorenz-Mie code (e.g. [31]) and then match the expected pattern with the experimentally observed pattern. The matching parameters are the transform matrix of physical drop position to pixel position on the camera chip (ABCD matrix).

Further details of this procedure can be found in [28]



Figure 14. a) Scattering angle calibration of the measurement plane using a traversing monodisperse droplet stream; b) Rainbow patterns of a monodispersed droplet stream at three different positions within the illuminated plane, taken separately but superimposed onto one image.

4. SUMMARY

This has been an intentionally brief review of selected recent developments in the field of particle/drop measurements in flows, because more comprehensive descriptions and analyses are available elsewhere. Nevertheless, each of the three techniques discussed, feature either extended measurement capabilities or improved measurement range/accuracy. For instance, the Depth from Defocus imaging technique increases the depth of field in resolving particles/drops and exhibits a well-defined detection volume, allowing reliable volumetric number densities to be computed; a capability which no other method presently offers adequately. The measurement of volume fraction of two-component drops is equally unique in its measurement capability, and incorporates an innovative approach to image processing, using a support vector machine (SVM). Finally, the planar version of rainbow refractometry extends conventional rainbow refractometry to an entire plane, a capability especially welcome when measuring in sprays.

None of the discussed techniques are yet to be offered commercially and there are certainly many limitations of each technique in application, which have not been thoroughly elaborated here. On the other hand, discussing these innovations may promote further developments and advances in measuring drops/particles in flows. No doubt the accuracy and reliability of each technique will also improve with advancing technology in hardware components, especially with respect to spatial and temporal resolution and sensitivity of imaging cameras.

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