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DYNAMIC PRESSURE PROPAGATION IN PIPES: MODELLING AND MEASUREMENT

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ABSTRACT

Several basic dynamical features of fluid-filled pipes are recalled. The fluid-wall coupling, the key factor governing the pipe dynamics, is outlined. The pressure propagation is shown to consist of a multitude of different waves, moving at different speeds. Both the low- and higher-order pressure waves are discussed. Attention is paid to the most important wave type associated with energy transfer: the plane wave. Several advanced techniques of pipe analysis concerning plane waves are discussed in some depth. These include 1) measurement of dynamic pressure and energy flow in the fluid using non-intrusive external wire sensors, 2) measurement of pressure pulsations at remote positions and 3) prediction of pressure pulsations within a coupled source-circuit assembly. Each technique is illustrated by examples obtained in real operating conditions.

Keywords: dynamic pressure, fluid-filled pipe, fluid waves, measurement, modelling.

NOMENCLATURE

B	$[Pa]$	fluid bulk modulus
E	$[Pa]$	Young's modulus
$\underline{\underline{I}}$	[-]	identity matrix
J	[-]	Bessel function
P	$[Pa]$	pressure amplitude
U,V,W	$[m]$	x-,θ-,r- displacement amplitudes
Z	$[Ns/m^3]$	fluid impedance
b	[-]	wave type indicator
d	$[m]$	mean pipe diameter
h	$[m]$	thickness of pipe wall
n	[-]	circumferential order
p	[Pa]	dynamic pressure
x,r	$[m]$	axial and radial coordinate
t	$[s]$	time
u,v,w	$[m]$	x-,θ-,r- displacement components
Δ	[-]	Laplacian operator
Γ	[-]	coupling factor
Ω	[-]	normalised frequency
Ξ	[-]	shell operator
α	[rad]	polarisation angle
ε	[-]	strain
φ	[rad]	phase angle
κ	[-]	normalised wavenumber
ν	[-]	Poisson's coefficient
θ	[rad]	polar angle
ρ	[-]	mass density
ω	[rad/s]	angular frequency

Subscripts and Superscripts

C	belonging to circuit
S	belonging to source
b	wave type index
c	cut-on
f	fluid borne
n	circumferential order index
s	solid borne (wall)
x, r, θ	axial, radial and polar directions

1. INTRODUCTION

A dynamically excited pipe propagates vibration along its wall and pressure pulsations within the contained fluid (gas or liquid). Due to the coupling between the fluid and the wall the axial component of propagation velocity is common to the two media. The velocity depends on the elasticity and mass distribution of both the fluid and the wall. A straight pipe will be considered having uniform properties in the direction of its axis.

Pressure pulsations propagate in an unbounded fluid at the speed of sound. Due to the coupling with the wall, the sound speed in the fluid decreases in comparison with the speed in the unbounded fluid. If the fluid is heavy (water, oil...) the decrease of sound speed can be quite substantial.

The fluid-wall coupling in pipes has been studied since more than a century ago. A simple coupling model was developed at the end of 19th century by Korteweg [1]. Zhukovskii published shortly after a study of water hammer propagation in pipes, [2]. In subsequent works one of the key issues was the modelling of pipe walls. Simplified formulae of sound speed for various external pipe conditions were published in papers [3,4], and books, [5,6].

The pipe motion becomes increasingly complex with the frequency rising. One of the earliest pipe models involving frequency dependence has been produced by Lin et al using a thin-wall theory applied to a fluid-filled cylindrical shell, [7]. It has been shown that the shell motion can be formulated in terms of waves of different frequency-dependent speeds of propagation. The cross section is then modelled in terms of natural modes while the motion in the axial direction is treated by the wave approach. Using such a dual formulation the response under a point source excitation located in the internal fluid has been found, [8]. The complex modelling has been further extended to the subject of pulsation and vibration energy, [9].

Basic pipe dynamic features are nowadays well understood. It became clear that a comprehensive analysis of pipe behaviour cannot fully rely on either computation or measurement alone. As a rule, the measurements have to be accompanied by adapted modelling if meaningful data are to be produced. The objective of this paper is to outline possibilities of hybrid, i.e. measurement + modelling pipe analysis and the conditions which allow for it.

2. ELASTIC WAVES IN A PIPE

At lower frequencies the pressure pulsations in a pipe dominantly propagate in the form of plane sound waves which travel along the pipe axis in opposite directions. However, at higher frequencies the propagation is far from being simple. The cross section may deform in a very complex manner and the distribution of pulsations may become intricate.

Pipe waves can be either propagating, quasi propagating or evanescent. The amplitude of the latter two types exponentially decreases from pipe discontinuities. The propagating waves dominate away from terminations or discontinuities.

Any wall deformation of the pipe cross section can be decomposed into circumferential harmonics of orders n. The deformation of the order n=0 is called "breathing mode", that of the order n=1 "flexural mode". Associated to each order n is a (theoretically infinite) number of propagating, quasi-propagating and evanescent wave types labelled by integer b. Thus each wave can be labelled by two indices: n and b, with n=0,1,2.. and b=1,2… Owing to the coupling each wave simultaneously coexists in the fluid and the wall. A wave of a particular type can propagate, and thus carry energy, only above a certain "cut-on" frequency which depends on the pipe and fluid properties. Below this frequency the wave is of evanescent or quasi-propagating type.

Three wave types of order 0 have zero cut-on, i.e. these waves can propagate at any frequency: the "pulsation", "extensional" and "torsional" wave. All other wave types, including the remaining ones of order 0, have non-zero cut-on frequencies. The torsional wave is fully decoupled from the fluid and travels entirely in the wall. The energy of the pulsation wave is largely confined to the fluid while that of the extensional one is largely confined to the wall. Besides the three mentioned wave types the n=1 "flexural" wave propagates at any frequency too. Its energy is essentially located in the wall.

It can be safely assumed that at frequencies up to the first cut-on pipe frequency only the pulsation wave effectively governs the dynamical pressure in the fluid. This wave is strongly coupled to the pipe wall which vibrates when subjected to fluid excitation. The pulsation wave is responsible for some key phenomena, such as water hammer.

3. THE PIPE MODEL

The pipe is usually modelled as a circular cylindrical shell filled with a homogeneous fluid, [7-11]. Fig. 1 shows the coordinate system, the pipe geometry and the wall displacements.

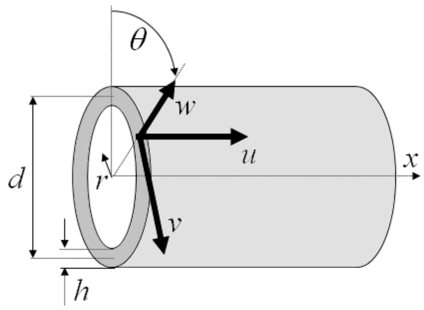

Figure 1. Coordinates and pipe displacements

As shown in [9] the coupled fluid-wall motion can be expressed using the equations of motion of the fluid, Eq. 1, that of the wall, Eq. 2, and the continuity condition at the fluid-wall interface, Eq. 3:

$$B\,\Delta p - \rho_f \frac{\partial^2 p}{\partial t^2} = 0 \qquad (1)$$

$$\left(\frac{4Eh}{d^2(1-\nu^2)} \varXi - h\rho_s \frac{\partial^2}{\partial t^2} \underline{\underline{I}} \right) \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ p \end{bmatrix} \qquad (2)$$

$$\frac{\partial p}{\partial r} + \rho_f \frac{\partial^2 w}{\partial t^2} = 0 \qquad (3)$$

In this work the 3×3 differential operator of wall elasticity, $\varXi$, uses Flügge's model, [12], but other models can be used too.

Assuming harmonic pipe motion with angular frequency ω=2πf the solution to Eqs. 1-3 can be found by factorization. This leads to a pressure pulsation distribution which is in the radial sense governed by Bessel function of order n, J_n, in the circumferential sense by cosine function of the same order and in the axial sense by the exp function. For any given n and b the spatial pressure distribution obeys:

$$p(r,\theta,x) = P\,J_n(k^r_{n,b} r)\cos\left(n\theta + \vartheta\right) e^{\mp jk^x_{n,b}x} \qquad (4)$$

The – and + signs in the exp term respectively denote the wave directions relative to the pipe axis (x axis). Here k^x and k^r stand for the axial and radial components of wavenumber k which make the vector sum of the two components equal to k:

$$(k^x)^2 + (k^r)^2 = k^2 = \omega^2 / c_f^2\,, \quad c_f = \sqrt{B/\rho_f} \qquad (5)$$

The three components of wall displacements of amplitudes U, V and W, satisfying Eq. 2 read:

$$\begin{bmatrix} u(\theta,x) \\ v(\theta,x) \\ w(\theta,x) \end{bmatrix} = \begin{bmatrix} U \\ V \\ W \end{bmatrix} \cos\left(n\theta + \alpha + \varphi\right) e^{\mp jk^x x} \qquad (6)$$

with φ=0 for u and w components and φ=-π/2 for v component.

By resolving Eqs. 1-6 a polynomial equation is obtained the roots of which lead to multiple solutions of k^r. Thus, a series of wavenumbers k^r and k^x are obtained for each wave type n,s and each frequency ω. Given any particular value of k^r the relationship between the radial displacement amplitude of the wall W and the fluid pressure amplitude P can be obtained in terms of a "coupling factor" Γ, Eq. 7:

$$\frac{P}{W} = \frac{4Eh}{d^2(1-\nu^2)}\Gamma, \quad \Gamma = \frac{\Omega^2 J_n(\kappa^r)}{\kappa^r\,\partial J_n(\kappa^r)/\partial\kappa^r} \qquad (7)$$

with the wavenumber and frequency expressed in a normalized (non-dimensional) form:

$$\kappa^r = \frac{k^r d}{2}, \quad \Omega = \omega\frac{d}{2}\sqrt{\frac{\rho_s(1-\nu^2)}{E}} \qquad (8)$$

The factor Γ represents the effect of coupling between the fluid and the pipe wall. This factor can be computed once the relationship between the radial wavenumber k^r and frequency ω is known.

For a particular pipe its wavenumber–frequency dependence relative to an order n and type b is called the dispersion law. It is usually presented in terms of normalized axial wavenumber κ^x=k^x d/2. Fig. 2 shows the dispersion diagram of a steel pipe of the 2.5% thickness-to-diameter ratio. The results are given for propagating waves only as these waves are of most interest in usual applications. The two groups of curves correspond to water-filled and empty cases. Empty pipes contain only 3 types of waves, b=1,2,3.

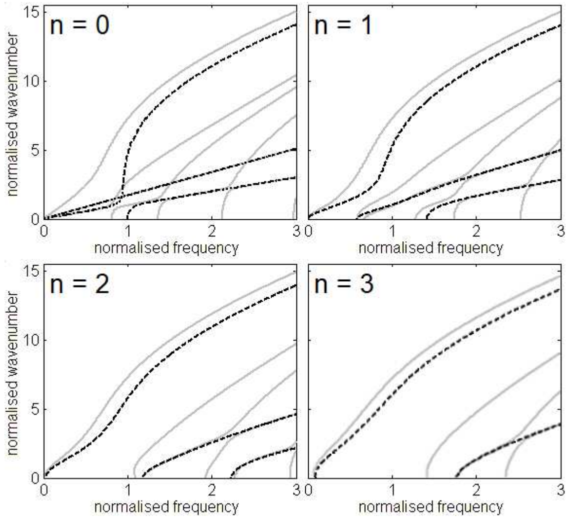

Figure 2. Dispersion diagrams for propagating waves in steel pipe. Full line: water filled; dashed line: empty.

The radial wavenumber component k^r defines the variation of pressure pulsations in the radial sense. Together with the order n it fully specifies the distribution of pressure in the cross section of a pipe due to the corresponding wave. In the same way the axial wavenumber component k^x together with the order n fully specifies the distribution of pressure pulsations in the axial sense. In addition to this the tandem k^x-n fully governs the distribution of wall vibratory displacement components u,v,w via Eq. 6 and thus the distribution of axial and tangential wall strains:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{\theta\theta} = \frac{2}{d+h}\left(\frac{\partial v}{\partial\theta} + w\right) \qquad (9)$$

The tangential strain $\varepsilon_{\theta\theta}$ is of particular meaning in this study as its measurement will be used for the detection of internal fluid pressure.

3. THE CUT-ON FREQUENCY

As mentioned in §2 a wave of a particular type can propagate and thus carry energy only above its own "cut-on" frequency. This transition frequency decreases with the pipe diameter but rises with its thickness. The cut-on frequencies increase with the increase of order n. Of a particular importance is the first cut-on frequency of n=2 order as this one has the lowest non-zero value. Below this specific frequency only 4 simple waves can propagate, as mentioned, and the analysis simplifies.

If a measurement is carried out below the lowest cut-on frequency a fairly basic sensor array suffices in view of only four wave types being capable to propagate. These are pulsating, extensional, torsional and flexural waves. In such a case the measurement can be done by employing piezo sensors to detect pressure pulsation and accelerometers to detect wall vibration. The latter can be configured in difference and sum schemes and oriented conveniently such to extract only one type of wave: either extensional, torsional or flexural. The problem may occur if the pipe diameter is large, resulting in a high frequency Ω, Eq. 8. For example, a 16mm thick steel pipe of 2m diameter has its first non-zero cut-on frequency of only 5.2 Hz if water-filled and of 10.5 Hz if filled with methane at 10b static pressure. Most of dynamic phenomena of such a pipe may involve propagating waves of higher-orders which would considerably penalise its analysis.

The analysis can be simplified if the selected sensor can itself respond to the wave of interest only. Where pulsation waves in a pipe are concerned these can be extracted out by using a strain sensor wrapped around the pipe. Such a sensor measures in fact the hoop strain due to breathing, i.e. the n=0 order. Still the pulsating wave is not the only wave type of the n=0 order. The extensional wave is of the same order too, but this wave generates weak hoop strains due to its dominantly axial motion. However, other waves of the n=0 order propagate at frequencies above the higher cut-on frequencies. The lowest non-zero cut on frequency of n=0 order defines thus the range of the effective measurement of pulsating wave using a hoop strain sensor. This frequency is considerably higher than the lowest cut-on of n=2 order. E.g. for a 2m/16mm steel pipe as mentioned above the lowest non-zero cut on frequency of n=0 order is 621 Hz if water-filled and of 281 Hz if filled with methane. Thus, by using a sensor which is sensitive to the n=0 order only the simplified analysis can be done within an extended frequency range.

For a pipe of given material and contained fluid the cut-on frequency normalised using Eq. 8, Ω_c, can be obtained as a sole function of the thickness-to-diameter ratio.

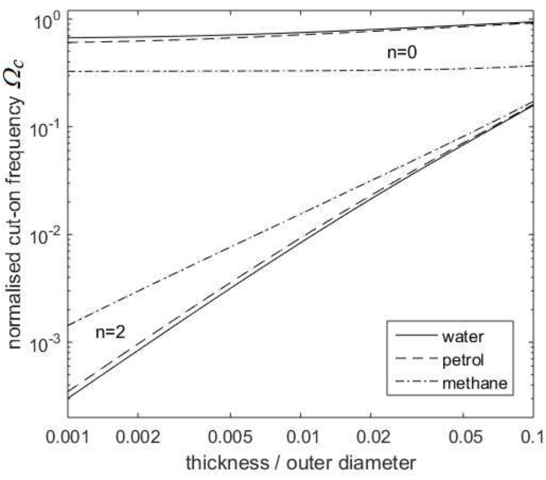

Figure 3. First normalised cut-on frequency of a steel pipe.

Fig. 3 displays the lowest non-zero values Ω_c of a steel pipe for three types of fluid. The difference between the lowest non-zero cut-on frequencies of n=2 and n=0 orders is seen to increase with the thickness/diameter ratio decreasing. The plot on Fig. 3 thus gives the upper frequency limit of a simplified experimental analysis of a straight pipe. The curves belonging to n=2 cut-on refer to the limit below which only the four basic waves can propagate. The n=0 curves refer to the basic breathing waves, the b=1 pulsating and the b=2 extensional wave, which can be measured using the integrating strain sensors as outlined above.

4. NON-INTRUSIVE MEASUREMENT

A piezo-electric wire wrapped around the pipe wall, demonstrated in [13], has been used in this study as a strain sensor. If firmly wrapped around the pipe wall, it directly measures the mean tangential strain around the circumference. This quantity can be related to the internal pressure using Eqs. 7 and 9 which account for the fluid-wall coupling. By applying one or several complete wraps around the pipe wall only the n=0 order of pipe motion is picked up, the other orders being suppressed by the integrating effect of the sensor. The advantage of using such a sensor is triple: it is non-intrusive, it is reusable and it filters out only the desired pipe deformation.

The wire produces an electrical charge which is proportional to its elongation. When connected to a charge amplifier the output voltage signal becomes thus proportional to the mean hoop strain. Fig. 4 shows the PVDF sensor mounted on a 1¼” steel pipe, a part of oil pumping circuit. In view of very high cut-on frequencies of this small pipe, 4.6 kHz for n=2 and 38.6 kHz for n=0, the measurement was safely carried out up to high frequencies.

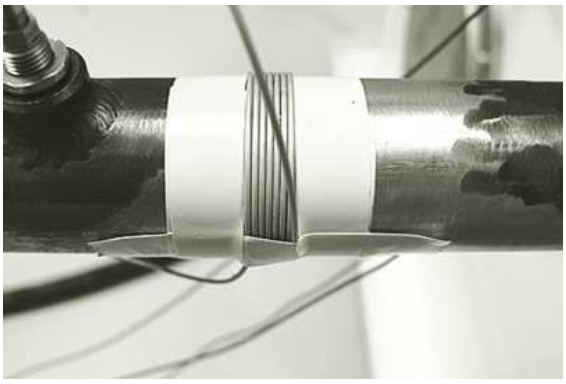

Figure 4. The PVDF wire sensor.

Fig. 5 compares the RMS levels of pressure pulsations obtained via the PVDF wire sensor and the classical intrusive piezo sensor. Quite remarkable matching between the two can be observed at not too high frequencies (top). At high frequencies (bottom) a discrepancy in levels becomes visible as some n=0 order waves other than the pulsation wave begin to propagate thus affecting the sensor output.

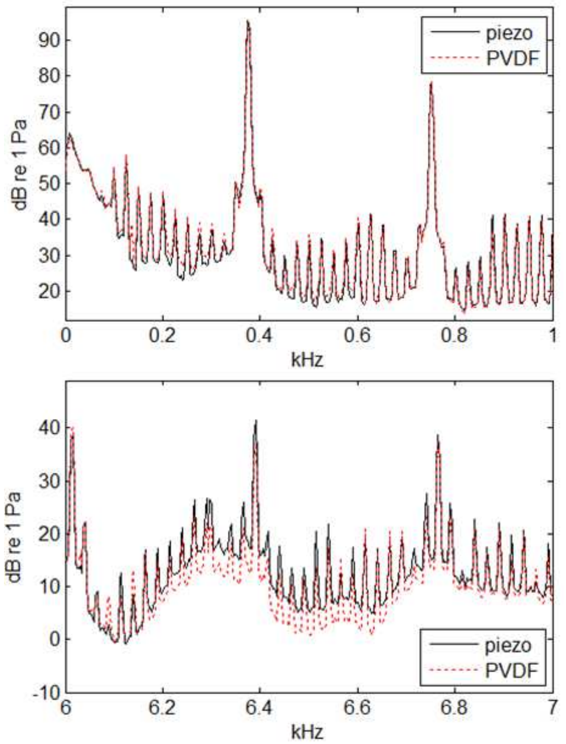

Figure 5. Comparison of pressure pulsation levels. Top: 0-1 kHz; bottom: 6-7 kHz.

5. SPEED OF PRESSURE WAVES

Advanced measurement analysis of pulsation waves, by using either intrusive pressure sensors or strain integrating sensors, allows a comprehensive pipe analysis. Such an analysis requires the precise knowledge of the actual wave speed in the pipe, which can itself be measured by an advanced technique. The following results can be obtained:

- velocity of pulsation waves (velocity of sound) in a given pipe
- flow speed using acoustical Doppler effect
- amount of damping in the fluid
- energy flow of pipe pulsations
- impedance of a hydraulic circuit
- reconstruction of pressure, vibration and strains along the pipe
- pipe thickness using vibration array
- characterisation of a source of pressure pulsation using a hydraulic "multi-charge" technique.

It has been shown that the velocity of sound can be obtained using a cost function computed from the frequency cross-spectra of measured signals, [14]. The method uses an array of 3 sensors. A modified two-parameter cost function can be obtained which indicates by its maximum both the sound velocity and the flow speed of the fluid. Fig. 6 shows the cost function obtained from a measurement on a pipe transporting compressed air: it indicates the speed of sound of 346 m/s and the flow velocity of 17 m/s.

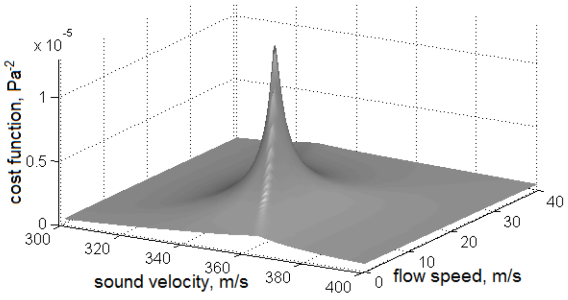

Figure 6. Cost function indicating the sound velocity and the flow speed.

6. APPLICATIONS

The knowledge of the true sound velocity in the pipe can be used to inversely compute the reference sound speed of the fluid itself and thus monitor the state of the fluid in operation. Any change in sound velocity of the fluid, produced e.g. by the presence of gas of impurities, will affect the sound speed and can thus be used as a warning to the operator.

The knowledge of true sound velocity can be further employed for the decomposition of wave components propagating along the pipe in opposite directions. The decomposed wave components can in turn be used for the reconstruction of pressure pulsations, vibrations and dynamical stresses at positions away from the measurement section. The same sensor array used for the measurement of wave speed can be employed to this end. If the pressure measurement is done by employing the external strain sensors, the reconstruction can be made for the strains and the corresponding stresses as well.

6.1. Intensity of pressure pulsations

Fig. 7 illustrates one of possible applications: measurement of energy flow of pipe pulsations. The energy flow per unit surface, the intensity, is shown as a function of frequency. The total intensity is then the frequency integral of the intensity spectrum shown. It is seen that the intensity measured using non-intrusive external sensors matches very well that obtained by classical sensors.

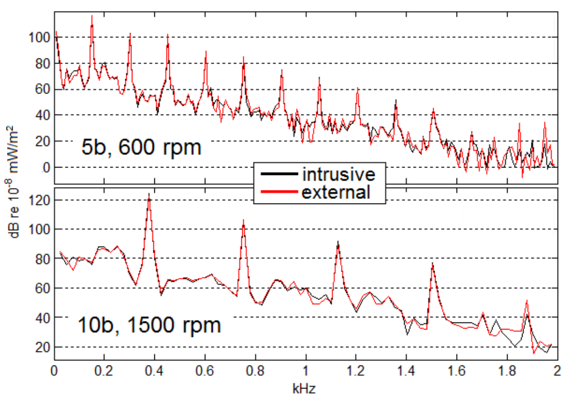

Figure 7. Intensity of pipe pressure pulsations. Top: 5b, 600rpm; bottom: 10b, 1500 rpm.

6.2. Remote pressure reconstruction

Fig. 8 shows the pressure pulsations measured directly and reconstructed from a 1.5m distant array of 3 sensors. The 6" 3mm thick pipe was filled with oil. The reconstruction was done by using the array data to decompose the pressure into positive- and negative-travelling wave components and by retro-propagating these components to the 4th remote point where the reference measurement was carried out in parallel. In this example intrusive piezo sensors were used. Very good matching is observed between the pressure measured directly and reconstructed; the mismatch shown by a grey line is relatively low. The processing of sensor signals was done in time domain using numerical filtering to account for the frequency-dependent coupling factor *Γ*.

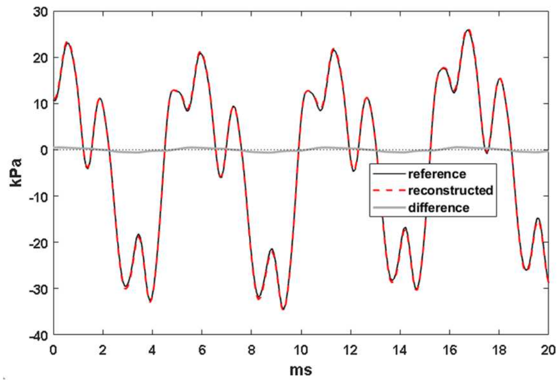

Figure 8. Comparison of pipe pressure pulsations measured directly and remotely.

The example shown in Fig. 8 was obtained in laboratory conditions. But the same technique was successfully applied to in-situ conditions too, e.g. on a dia 2m penstock of a hydraulic power plant, using strain PVDF sensors. Moreover, it has been shown in [15] how the pressure profile along an entire pipe can be reconstructed using the reconstruction technique.

6.3. Source characterisation

Owing to the technique of wave decomposition a pipe can be used for the characterisation of sources of pressure pulsations, such as pumps, compressors, hydro-motors etc. Analysis of hydraulic components by modelling and measurement has been in use since a long time ago. In [16] the authors have made a review of several available analysis techniques.

If the pipe conducts effectively only plane waves, as is often the case, a simple relationship can be established in frequency domain between the pressure at the outlet of the source, p_C, and the intrinsic pressure of the source p_S. The latter denotes the source pressure which would have been produced had the outlet been shut. When the coupling between the source and the circuit is strong, the coupled pressure p_C could be quite different from the intrinsic source pressure p_S. Expressed in terms of frequency pressure amplitudes *P*, the relationship between the two pressures depends on the impedance of the hydraulic circuit at the coupling position, Z_C, and the impedance of the source at the same position, Z_S:

$$P_C = Z_C / (Z_S + Z_C)\, P_S \qquad (10)$$

Eq. 10 shows that the source is characterised by two parameters: its intrinsic pressure, P_S, and its own impedance, Z_S. By connecting the source to two or more circuits of different impedances these two quantities can be retrieved from the measurement data. This characterisation principle has been first conceived for application on engine exhaust lines, [17-19], but can be readily extended to hydraulic sources [20]. The circuit impedance can also be modelled using the impedance continuity principl.

Fig. 9 shows the comparison between the levels of pressure pulsations in an oil circuit obtained by direct measurements and by prediction. The latter used the separately characterised source pressure and impedance and applied to Eq. 10. The source was a gear electro-pump producing periodic pulsations which are consequently represented by the associated harmonic indices 1, 2…. The total levels of pulsations are indicated by the right column. The circuit impedance Z_C needed for the characterisation was measured by the technique outlined in [21]. A fairly good matching of the measured and predicted results can be seen.

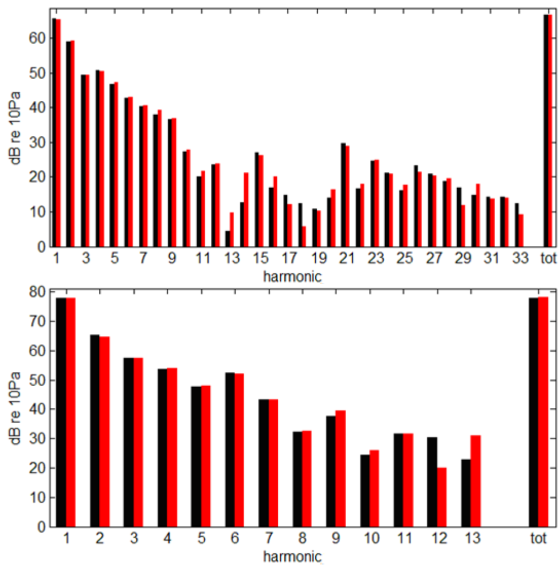

Figure 9. Comparison of levels measured directly (in black) and using Eq. 10 (in red). Top: 5b, 600 rpm (first harmonic 150 Hz); bottom: 10b, 1500 rpm (first harmonic 375 Hz).

7. CONCLUSIONS

A fluid-filled pipe exhibits a complex dynamic behaviour which can be conveniently analysed using multi-point measurements combined with analytical modelling. The motion in any cross section of the pipe is a sum of circumferential orders *n*=0,1,2… which govern the deformation of pipe wall and the dynamic pressure distribution within the pipe. To each of these orders is associated a number of pipe waves of different kinds, each having its own wave speed and pressure distribution.

Where the pressure pulsations are concerned one of the waves of the order *n*=0, called the pulsation wave, is of a particular importance as it transports most of the energy of pressure pulsations along the pipe. An external, non-intrusive strain sensor made of piezo wire was shown to provide a simple means of the measurement of pulsation waves.

The hybrid approach of pipe analysis, i.e. the multi-sensor measurements of pressure pulsations combined with fluid wall modelling, enable one to carry out advanced pipe analysis. Several examples of such analysis based on measurements in operating conditions are outlined in the paper: measurement of fluid energy flow using wire sensors, measurement of pressure pulsations at remote positions, prediction of pressure pulsations of a coupled circuit. These examples show the potential of hybrid approach for prediction, monitoring and diagnostic purposes.

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