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MODELING (UNDERSTANDING AND CONTROLLING) TURBULENT FLOWS: THE HERITAGE OF LEONARDO DA VINCI IN MODERN COMPUTATIONAL FLUID DYNAMICS

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"Quando tu metti insieme la scienza de' moti dell'acqua,
ricordati di mettere di sotto a ciascuna proposizione li sua giovamenti,
acciò che tale scienza non sia inutile", Codice Leicester, Leonardo da Vinci, 1506-10

"What remain of a man are the dreams we linked to his name", P. Valery, 1894

ABSTRACT

Why it is possible to claim that Leonardo da Vinci has been the "inventor" of the scientific method decades before the Ones (i.e. Galileo Galilei for instance) the History of Science is traditionally giving the fatherhood?

Why Leonardo da Vinci is (somehow) an *ante-litteram* fluid-dynamic scientist?

Why Leonardo's approach can be considered an anticipation of modern applied physics (CFD) and why his newness has not yet fully appreciated?

Taking the move from the above three questions, the present work possibly explores the open literature to find proofs of Leonardo's contribution to modern fluid dynamics. The manuscript focuses on three pillar contributions chosen, in the vast repertoire of Leonardo's Notebooks and Artworks, to give a personal perspective on his contribution to the frontiers of the fluid dynamics investigation. Specifically, the manuscript advocates: the link between flow visualization and modern deep learning usage in flow modelling (Section 2), the eco-design perspective implicit in the mimicry of Nature (Section 3), and the intuition of a science of quality and patterns (Section 4).

1. INTRODUCTION

Why it is possible to claim that Leonardo da Vinci has invented the scientific method decades before the ones the History of Science is traditionally giving the fatherhood? Why Leonardo da Vinci is (somehow) an *ante-litteram* fluid-dynamic scientist? Why Leonardo work can be read as an anticipation of modern applied physics?

This three-question-incipit addresses the motivation behind the significance of Leonardo da Vinci to the applied sciences (in general) and the fluid dynamics and related engineering (in detail).

Leonardo da Vinci was, in today's scientific jargon, a systemic thinker with a nearly irrational interest in finding an interpretation of Nature [1]. To describe his life, using the modern metaphor of leadership, we can adopt Sinek's words "If we were all rational, there would be no exploration, there would be very little innovation and there would be no great individuals to inspire all those things. It is the undying belief in something bigger and better that drives that kind of behavior" [2].

Leonardo's scientific work was virtually unknown during his lifetime and as such remained for nearly two centuries after his death in 1519. Moreover, the novelty of his approach to the understanding of Nature is still to be fully disclosed and represents a topic of interest for a number of scholars [3]. In the era of the *Divina Proportione* (i.e. Divine (or Golden) Ratio, 1509), the mathematic book authored by Luca Pacioli and illustrated by him, Leonardo was following the idea of Nature investigation by analogies. Understanding Nature by looking at hidden connections among phenomena, through a similarity of patterns: interlinking animal physiology and engineering, patterns of turbulence in water and in the flow of air, and from there the exploration of sound, the theory of music, and the design of musical instruments. Notably, he progressed through analogies and interconnecting patterns, similar to the so-called archetypes as in the early stage of evolution theory (C. Darwin) or cybernetic era (G. Bateson). His learning and research process was then genuinely multi-disciplinary. Without the use of Euclidean geometry, far from the advent of Newtonian physics, He was in need of a new kind of qualitative mathematics now formulated within the framework of complexity theory [1]. The Leonardo mathematics was made of visualization, unveiling the topology and the geometry in motion of phenomena.

1.1. The method

Five hundred years before the scientific method was recognized and formally described, Leonardo da Vinci developed and practiced its essential flow chart — study of the available literature, systematic observations, careful and repeated experimentations, formulation of theoretical and generalized models. The famous manifesto on his scientific method claimed:

But first I shall do some experiments before I proceed farther,
because my intention is to cite experience first and then with reasoning
show why such experience is bound to operate in such a way.
And this is the true rule by which those
who speculate about the effects of nature must proceed.

Notably, in the intellectual history of Europe, Galileo Galilei, who was born 112 years after Leonardo, is usually credited with being the first to develop this rigorous empirical approach.

The empirical approach came to Leonardo because of his revolutionary change brought to natural philosophy in the fifteenth century, driven by his relentless reliance on direct observation of nature, complemented by visual memory and drawing skill. Repeatedly belittled by the Greek philosophers and scientists tradition. Never tired of emphasizing the importance of *sperienza*, the direct experience of natural phenomena as given in many declarations. To mention but a few: “All our knowledge has its origin in the senses” (*Codex Trivulzianus*) [4], “Wisdom is the daughter of experience,” (*Codex Forster*) [5], “To me it seems that those sciences are vain and full of errors that are not born of experience, mother of all certainty” (*Trattato della Pittura*). These comments were used to address also methodological issues on the good practice to conduct experiments, stressing in particular their careful repetitions and variations. *Manuscript A* [6]: “Before you make a general rule of this case, test it two or three times and observe whether the tests produce the same effects.” *Manuscript M* [7]: “This experiment should be made several times, so that no accident may occur to hinder or falsify the test.”

The final methodological step, in Leonardo speculations, was the introduction of simplified models to distillate the essential features of complex natural phenomena. For instance, he represented the flow of water through a channel of varying cross sections by using a model of rows of men marching as it is now customary in crowd dynamics modelling [8].

1.2. The power of visualization

The preferred tool of analysis was Leonardo drawing capability and the promptness of his vision, giving him the possibility of merging in one action the act of observing with that of documenting. Drawing was then the intellectual vehicle to formulate conceptual models in a geometry language for his organic forms science. Drawing was the link among art, design (in an engineering sense), and science.

From a methodological viewpoint, Leonardo called his drawings “demonstrations”, as customary in mathematics, claiming that they gave “true knowledge of (various) shapes, which is impossible for either ancient or modern writers … without an immense, tedious and confused amount of writing and time”. Such a graphical approach is the language of symbolic analysis in modern engineering.

From a technical viewpoint, Leonardo introduced the practice of preparatory drawings, using whom he was sketching several alternative (dynamic) descriptions of the phenomenon under scrutiny. These preparatory sketches have the quality of trying to visualize the dynamic quality of Nature (e.g. in a similar fashion to early twentieth century Futurist theory of photography and Bragaglia’s *Photodynamism* experiments [9]. Following Capra [1], this visual technique can be linked to the emergence of qualities (of the observed phenomenon) out of chaos and confusion. The emergence defined in the complexity theory, which is one of the key property of living nature.

Leonardo’s science is utterly dynamic, because he realizes that living forms are continually transformed by underlying processes. Every portrait of the world is then only one configuration in a continual process of transformation.

1.3. The eco-design

The notion of design as a distinct discipline emerged only in the twentieth century, notwithstanding Leonardo can be seen as a designer where design should be intended as the broad process of giving form to objects.

At its outset, the design process is purely conceptual, involving the visualization of images, the arrangement of elements into a pattern in response to specific needs, and the drawing of a series of sketches representing the designer’s ideas. Good designers have the ability to think systemically and to synthesize. They excel at visualizing things, at organizing known elements into new configurations, at creating new relationships. What made Leonardo unique as a designer and engineer, however, was that many of the novel designs he presented in his Notebooks involved technological advances that would not be realized until several centuries later. His design process moved from the understanding of natural models as customary of the ecological design routed in bio-mimicry concepts. In this respect, he was the one leading the transition from engineering to science, from know how to know why.

Leonardo's science is a science of qualities, of shapes, of correlations, rather than absolute quantities. He preferred to *depict* the forms of nature in his drawings rather than *describe* their shapes, and he analysed them in terms of their proportions rather than measured quantities.

To describe Leonardo's contribution to modern fluid dynamics in the following Sections the original questions find (possibly) a response in terms by categorizing three main contributions of Leonardo to modern fluid dynamics. Namely, Section 2 discusses the works of Leonardo under the viewpoint of flow visualization and flow modeling. Section 3, then, illustrates the activity of Leonardo as eco-designer inspired by Nature. Last, Section 4 suggests the role of Leonardo in anticipating the modern science of patterns by his intuition on so-called "geometry done with motion".

2. DEMONSTRATIONS BY DRAWINGS, THE FLOW VISUALIZATION

Leonardo was fascinated the manifestation of fluids, water first and then air. As He did found in fluids their relevance to life, being the matrix of all organic forms. The objective behind such interest was mainly in the role of fluids in natural eco-systems he deepened by observing flows of rivers and tides, mapping of entire watersheds, and currents in lakes and seas, flows over weirs and waterfalls, and the movement of waves as well as flows through pipes, nozzles, and orifices.

Leonardo recognized that fluid (water) dynamics is governed by two principal forces, the force of gravity (mass) and the fluid's internal friction (surface), or *viscosity*, and he correctly described many phenomena generated by their interplay. He also realized that water is incompressible and that, even though it assumes an infinite number of shapes, its mass is conserved. Leonardo extended his keen interest in friction to his extensive studies of fluid flows. The *Codex Madrid* [10] contains meticulous records of his investigations and analyses of the resistance of water and air to moving solid bodies, as well as of water and fire moving in air. Well aware of the internal friction of fluids, known as viscosity, Leonardo dedicated numerous pages in the Notebooks to analyzing its effects on fluid flow. "Water has always a cohesion in itself," he wrote in the *Codex Leicester* [11], "and this is the more potent as the water is more viscous".

In the second center of gravity of Leonardo's investigations, lie vorticity and turbulence. Throughout the Notebooks, there are countless drawings of eddies and whirlpools of all sizes and types. Taken from the observations of currents in rivers and lakes, behind piers and jetties, basins of waterfalls, or behind objects of various shapes immersed in flowing water. These drawings witnessed the fascination of Leonardo with the ever-changing and yet stable nature of this fundamental type of turbulence. This fascination is the result of a deep intuition that the dynamics of vortices, combining stability and change, embody an essential characteristic of living forms [1].

Leonardo was the first to understand the detailed motions of water vortices, and distinguished between flat circular eddies (with solid body rotation), and spiral vortices (or free vortices) that form a hollow space, also called funnel, at their center. "The spiral or rotary movement of every liquid," he noted, "is so much swifter as it is nearer to the center of its revolution.

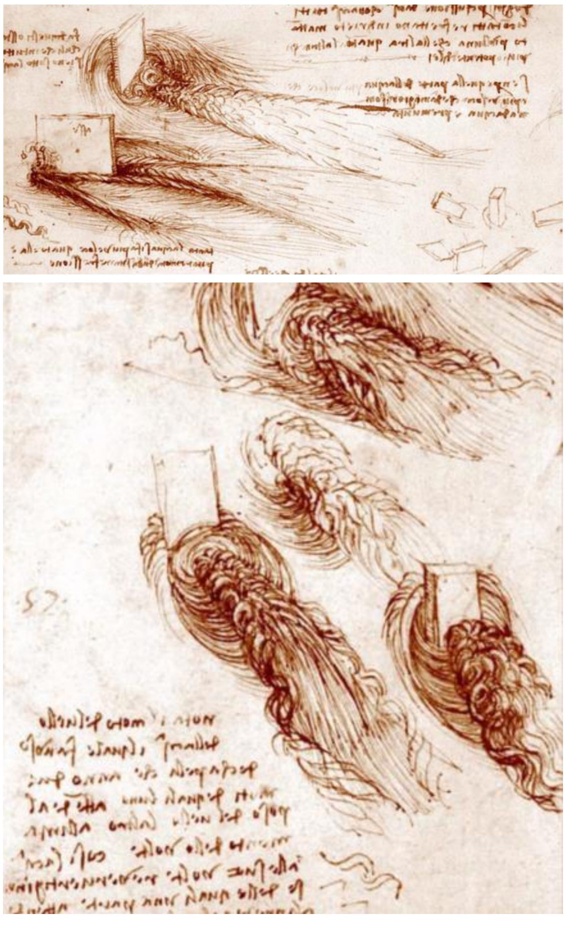

Figure 1. Windsor Collection, Landscapes, Plants, and Water Studies, 1509-1513 [12]

Figure 1 shows drawings from the Windsor Collection [12], Landscapes, Plants, and Water Studies, (1509-1513), depicting the observations (demonstrations) about vortex shedding behind a rectangular plank (i.e. varying what we called the water angle of attack) and a blunt body.

Apart from the speculative goal of his observations, driven by the objective of designing hydraulic works, it is remarkable to find the visualization of vortex structures which are typical of flow configuration pertinent to turbomachinery fluid dynamics. The next Figure 2 shows, as an example, the prediction of horse-shoe vortices at the root of a blunt blade [13] and tip leakage vortex formed in a compressor cascade [14].

Such detailed studies of vortices in turbulent water were not taken up again for another 350 years, until the analysis of vortex motion authored by Hermann von Helmholtz in the mid-nineteenth century.

As far as the method is concerned, Leonardo’s scientific drawings, whether they depict elements of machines, anatomical structures, turbulent flows of water, or botanical details, were never limited to a realistic representation of a single observation. But, they were syntheses of repeated observations, crafted in the form of theoretical models.

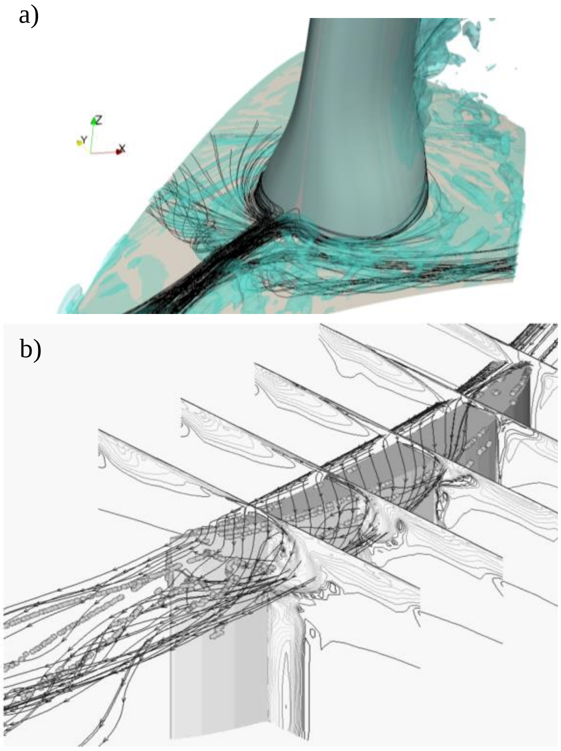

Figure 2. Examples of CFD simulated vortex structures in turbomachinery pertinent configurations: a) horse-shoe vortices at the root of a blunt blade [13], and b) tip leakage vortex in a linear compressor cascade [14]

To obtain the theoretical model, observation needed to be organised and represented in an explicit and intelligible form. Mathematic and statistic approaches then followed, leading to the final model. However, regardless of the scientist ability and creativity, this approach has always been restricted to a limited number of observations, a few thousands at maximum, due to the incredible complexity of the problems in analysis.

Today, while the basis of the learn-by-draw approach remain the same, technological innovation has equipped scientists with powerful and revolutionary tools. The growth in available computational power and the capability of directly deriving thousands of observations in a digital form, either from numerical simulations or laboratory instruments, has led to a dramatic increase of accuracy and robustness of the derived models.

A well-fitting example of this progress regards the study of turbulent flows. Realizing the idea of Leonardo da Vinci, i.e. to recognise frequent patterns in hundreds of turbulent flows in order to derive simple mathematical laws, is now reality thanks to the use of *machine learning* tools. In the past five years, a small but growing community has applied machine learning to create alternative but still reliable data-driven turbulence models. They all share a common feature: they try to overcome the restriction imposed by the well-known Boussinesque closure in a RANS approach.

While this cutting-edge studies still present minor issues, especially in term of generality and computational cost requirements, in many cases they have strongly enhanced the standard turbulence modelling, such for example one of the first presented work on the topic from Tracey *et al.* [15]. An enhanced Spalart-Allmaras model was derived, more accurate and closer to the physical behaviour of the flows, even if at high computational costs and viable for very simple geometries only. Zhang and Singh [16], which have improved the model in generality and they were able to correctly reproduce turbulent flow in simple configurations. In the same year, Ling *et al.* [17] applied multilayer perceptron neural networks to predict a turbulent flow in a specific configuration. Galilean invariance of the solution was granted by the use of turbulent invariants.

Remarkable efforts have been spent on achieving a better near wall modellization, in comparison standard RANS approach. This has been obtained through a data-driven derivation of the anisotropic shear stress tensor. In 2017 Wang *et al.* [18] used statistics obtained from DNS simulations to train a multilayer artificial neural network. A zonal training was also used, allowing the reduction of redundant information in the training dataset. The model performed better that the standard k-epsilon for two test cases.

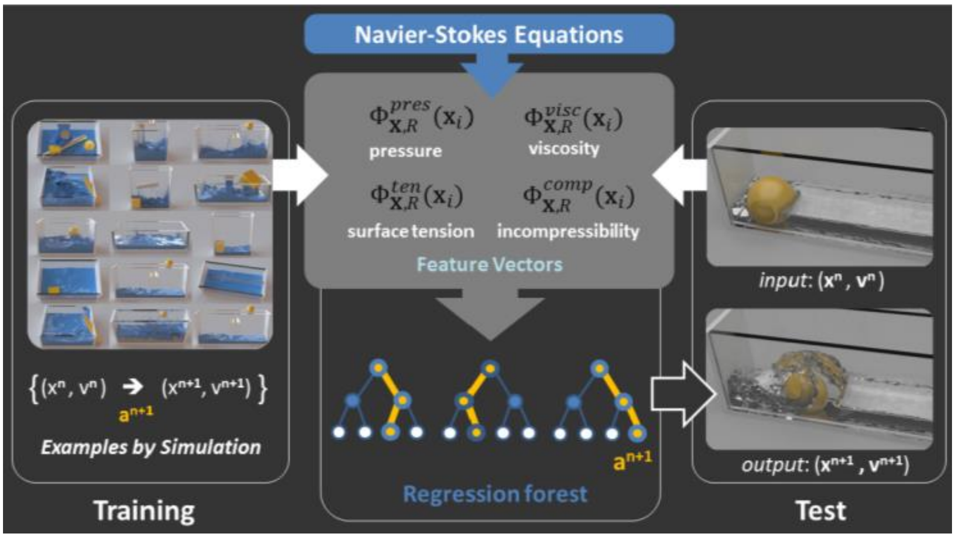

Figure 3. General concept of machine learning approach applied to fluid dynamics [20]

Machine learning has also been proven to be extremely reliable for complex internal flows, even in presence of heat transfer, as highlighted by Sandberg *et al.* [19]. A response surface methodology (RSM) was successfully applied to derive the anisotropic part of stress tensor, enhancing the prediction capability of k-v^2-ω turbulence model for film cooling on turbine blades. A similar approach was used to derive wake vorticity on rotor blades of a low pressure turbine.

Novel machine learning based approach, have been also proposed to formulate physics-based fluid simulation as a regression problem, in multi-phase flow simulations [20].

This approach designed a feature vector, directly modelling individual forces and constraints from the Navier-Stokes equations to solve high-resolution fluid simulations in computer graphic scene with millions of particles. **Hiba! A hivatkozási forrás nem található.** shows a general framework for the implementation of machine learning into fluid dynamics based upon the experiences of Ladicky and co-workers [20] on the use of data driven regression approaches.

3. BIO-MIMICRY, THE ECO-DESIGNER

Leonardo's science was a gentle science [1]. In accordance with his organic interpretation, instead of finding routes to dominate nature, as advocated by Natural Science since the seventeenth century, Leonardo's intent was to learn from her as much as possible.

The discovery of the complexity of natural forms, patterns, and processes, gave Leonardo the awareness that nature's ingenuity was far superior to human design. He declared "(humanity) will never discover an invention more beautiful, easier, or more economical than nature's, because in her inventions nothing is wanting and nothing is superfluous". Nature is a model and a mentor in the vein of modern ecological design or eco-design practice. Like Leonardo da Vinci five hundred years ago, eco-designers today study the patterns and flows in the natural world and try to incorporate the underlying principles into their design processes.

Figure 4 shows a drawing from the Codex on the Flight of Birds (1505) [21] with the study of a mechanical wing as imagined to power a human-powered flying machine.

Figure 4. Study of a mechanical wing imitating a bird wing, Codex on the Flight of Birds, 1505 ca. [21]

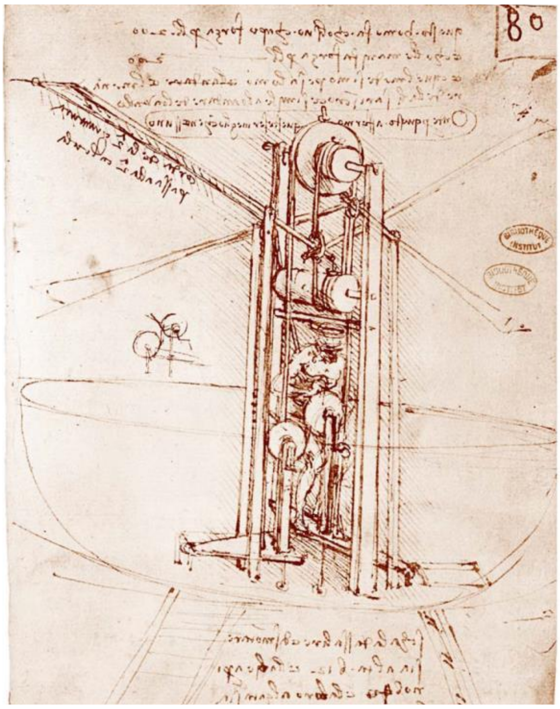

Figure 5. Leonardo's flying ship, Ms B, 1487-90

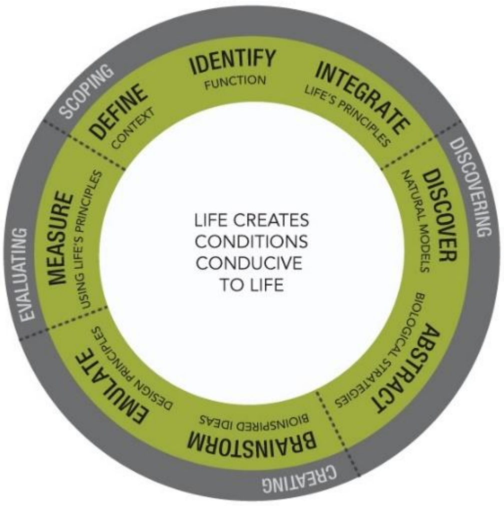

Figure 6. Eco-design circle in bio-mimicry [23]

Air resistance was of special interest to Leonardo, because it played an important role the flight dynamics of birds and, consequently, in the possibility of distillating concepts leading to the design of flying machines. He noted in the *Codex Atlanticus* [22] "In order to give the true science of the movement of birds in the air it is necessary first to give the science of the winds" (1478-1519).

He found that the air under a bird's wing is compressed by the downstroke: "See how the wings, striking against the air, sustain the heavy eagle in the thin air on high". From this observation He than concluded "As much force is exerted by the object against the air as by the air against the object." Leonardo's finding was re-stated by Isaac Newton two hundred years later and has since been known as Newton's third law of motion.

As illustrated in Figure 5, Leonardo is ultimately using a bio-mimetic approach in his design of flying machines which is based on the intuition of distillating from the observation of Nature the "working pinciples" to be modelled. Implicitly introducing, with hundreds years of anticipation, the eco-design circular methodology shown in Figure 6.

Biomimicry is still very important today in fluid dynamics, as turbulence is one of the few problems of classic physics still waiting for a breakthrough. This is in part due to the fact that defining turbulence is per se tricky, as everybody can recognize the difference between a turbulent and a non-turbulent flow, yet all the possible definitions one can find in literature are indirect and based on said difference. On the other hand, at this day, the only solutions of turbulent flows come from Direct Numerical Simulations of the Navier-Stokes equations, that are affordable only for moderate Reynolds numbers and simplified geometries.

Most of the technological advancements in fluid dynamics came from empirical experimental experience and, later on, from modelling of Navier-Stokes equation. Here "modelling" is a politically correct word that conceals a violent story of deliberately neglecting most of the physics of turbulence to come up with the washed out Reynolds-Averaged version of Navier-Stokes equations to reduce costs in terms of time and computing power. With all the due differences, it is a condition similar to that experienced by Leonardo during his life, when he was observing flow phenomena he could not mathematically model but only try to mimic in his inventions. This is a strategy that is still valid and is now known as biomimetics or biomimicry: the examination of nature, its models and processes to take inspiration for or to emulate in man-made designs. As a matter of fact, evolution provides us with plenty of aerodynamics solutions optimized through natural selection over hundreds to million of years, that allowed to animalia and plantae to adapt, survive and develop.

For example, trees have developed very efficient ways to grow maintaining their trunks strong and flexible to withstand adverse weather actions [24], to disperse their seeds for very far distances in order not to shade children-trees [25], Figure 7. Flyers and swimmers, on the other hand, have developed low-drag surfaces or high-efficiency wings and fins, often leading to solutions that are technologically impossible to replicate [26]. Just try to imagine the difficulties in building a mechanical bat, with its flexible wings, and over 10 different mobile junctions necessary to reproduce its complex flapping [26]. However, despite the difficulty inherent in the approach, the insight biomimicry provides is a source of inspiration for possible technical solutions. Further, evolution acted in this case as an optimization procedure, that favoured some solutions and eliminating the less efficient ones through natural selection.

Figure 7. Maple seed (Wikipedia)

Here follow three examples, one to reduce drag inspired by the skin of sharks, one to control separation and stall, inspired by humpback whale pectoral fins and finally one to reduce noise, inspired by the owl feathers.

3.1. Superhydrophobic low-drag surfaces

The most basic forms of drag are pressure drag and friction drag, the former associated with the energy required to move fluid from the front of an object, around it and then back behind, the latter to friction with the body surface. While pressure drag can be reduced by streamlining the shape of the body, friction drag is associated to the surface of the body and the evolution of fluid along said surface. In particular, increasing the fluid velocity leads to the formation of streamwise vortices that develop along the body surface in streamwise direction, increasing the turbulence of the fluid and triggering laminar to turbulent transition.

A possible way of controlling the formation of these vortices was suggested by the study of the skin of sharks, that is characterized by a pattern of riblets, Figure 8.

Scholars associated these patterns to their capability of controlling the evolution of streamwise vortices [29]. It was also found that this capability is associated to the shape of the riblets and the relative size of riblets and the streamwise vortices to be controlled. In fact the drag-decreasing effect occurs only when the vortices are larger than the riblets and therefore stay above them, and the flow inside the riblets valley is sufficiently calm. This leads to a viscous sublayer characterized by low values of turbulent kinetic energy and fluctuations and an outer layer with values similar to those of a smooth flat plate. When the velocity increases and the size of the vortices decreases down to values smaller than the riblets spacing, the overall effect is to increase the drag of the surface by energizing the viscous sublayer [29].

A number of different patterns was investigated, mimicking different shark skin patterns and still some with highly three-dimensional patterns are under scrutiny [30].

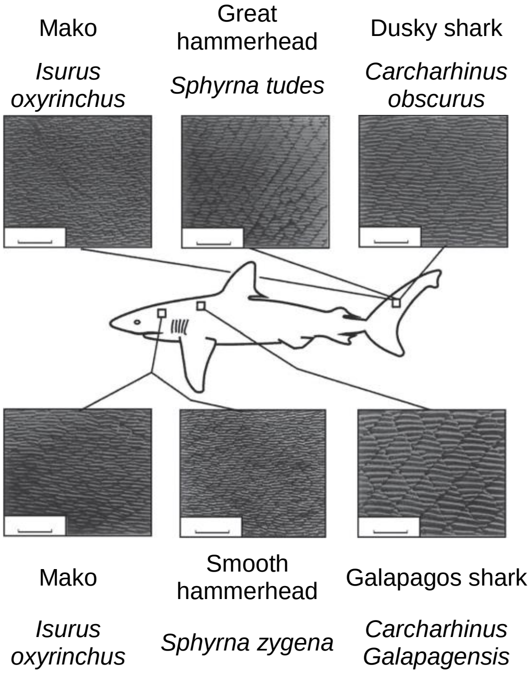

Figure 8. Scale patterns of fast-swimming sharks. Scale bar 0.5 mm [27, 28]

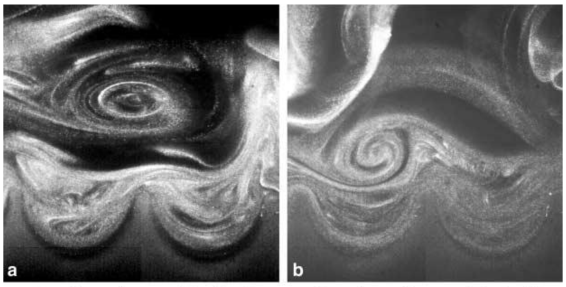

Figure 9. Flow visualization of streamwise vortices on cross-sections for drag decreasing case a) and drag increasing case b), respectively corresponding to a shark swimming velocity of 3, 5 m/s [29]

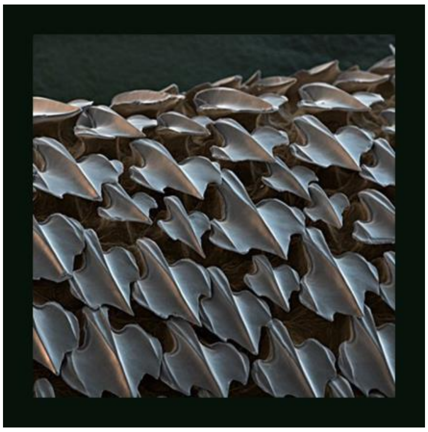

Figure 10. Shark skin magnified. These dermal dentacles are sharp structures. (NOAA Photo Library)

3.2. Leading edge bumps for trailing edge separation control

Despite the promise of nature-inspired solutions to turbomachinery stall control, most are not effectively useful in aerospace or turbomachinery applications as the route of evolution in maximising lift and minimising drag occurred over a range of Reynolds numbers that are simply too low. One of the few exceptions to this is found in the humpback whale (*Megaptera novaeangliae*), as it is one of the few animals that can provide information relevant to high-Reynolds number applications. This particular mammal is able to perform sharp rolls and loops under water whilst hunting, Figure 11. Marine biologists attribute this capability to the peculiar shape of its flippers, Figure 12, characterised by a wing-like aspect ratio and a wavy leading edge with typically ten or eleven rounded tubercles. Tubercles have been associated with the same mechanism that is induced on the flow field by aircraft strakes: the capability to keep the boundary layer attached over the wing and in so doing maintaining lift at higher angles of attack (AoA). The tubercles therefore act as a stall-control system.

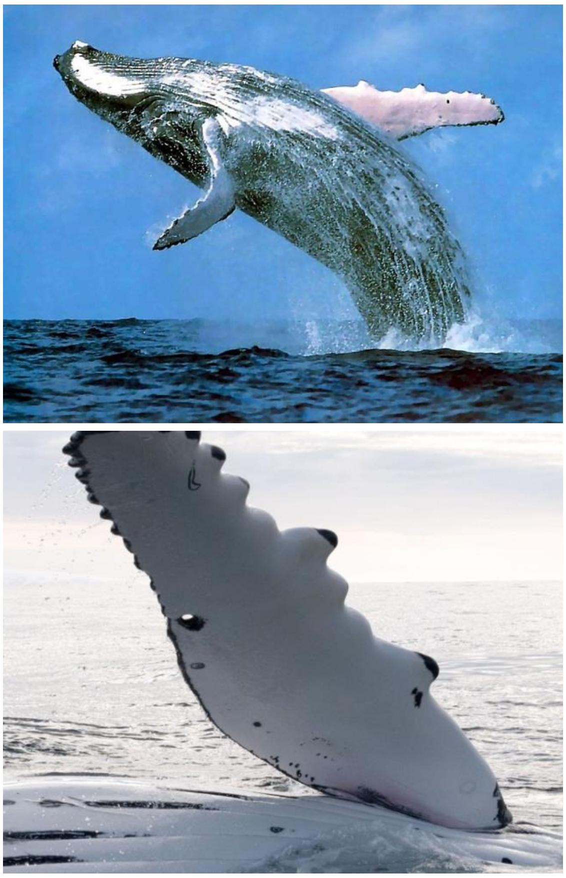

Figure 11. Humpback whale (up); detail of the whale pectoral fin or flipper with its tubercles (bottom)

Among the pioneering studies on biology-based concepts for hydrodynamics, Fish [31] analysed the efficiency of swimming mammals highlighting the drag reduction mechanisms that developed with the evolutionary process. Fish *et al.* in [32] discussed passive and active flow control mechanisms in natural swimmers and their technological exploitability. Hua *et al.* [33] carried out a biomimetic numerical study on stall limit to establish the performance of a seagull wing specifically studying the role of their natural camber. Notably, this investigation demonstrated the advantages of a naturally cambered wing by comparing its lift-to-drag ratio against a NACA four digit airfoil.

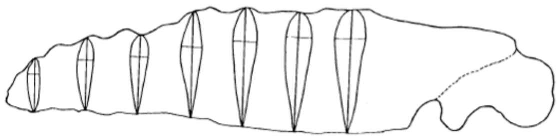

Figure 12. Planform of a humpback whale pectoral fin, after Fish et al. [34]

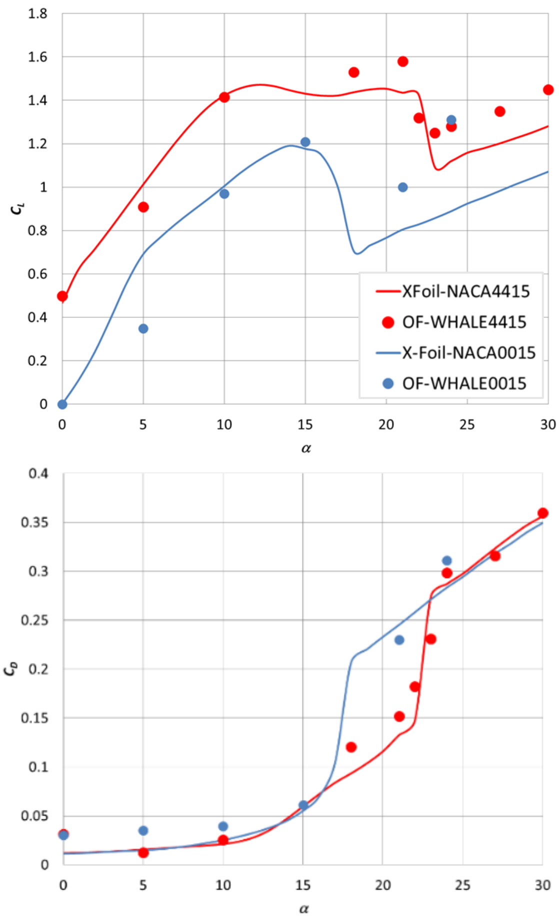

Figure 13. Lift (left) and drag (right) coefficient vs AoA. WHALE0015 and WHALE4415 identify the sinusoidal leading edge blades

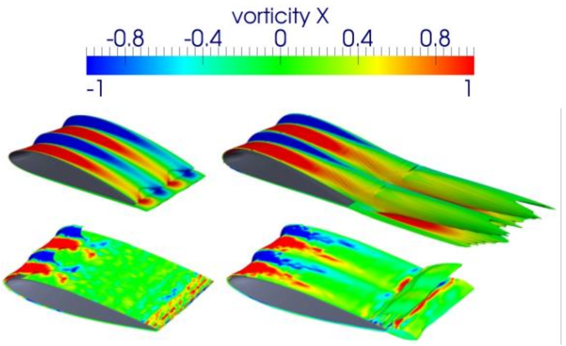

Figure 14. A view of suction side with enstrophy isosurfaces: AoA 10deg (left) and 21deg (right) for cambered (top) and symmetric (bottom) airfoil

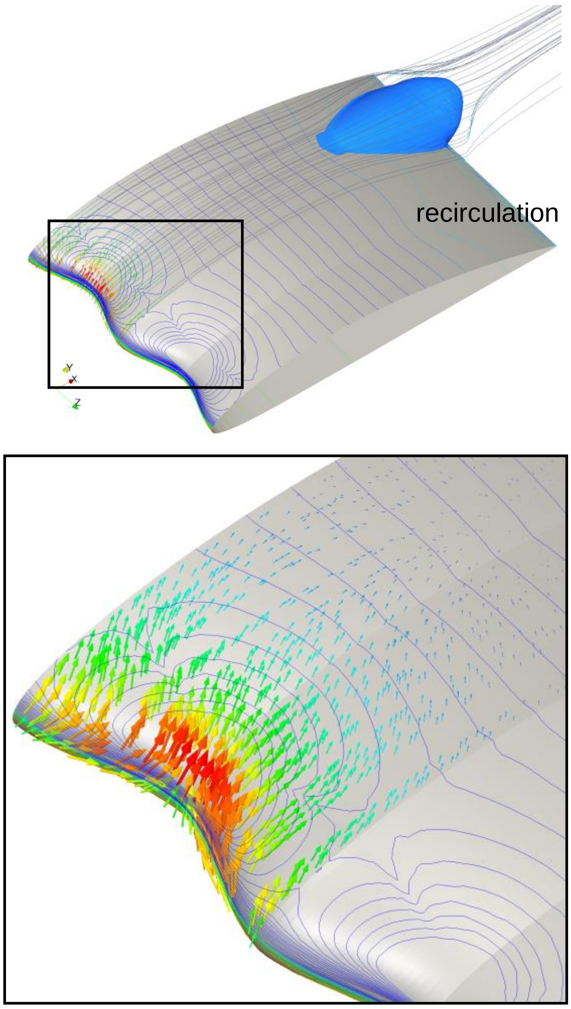

Figure 15. WHALE4415 (AoA=21deg): an insight on the distortion of the velocity field generated by the leading edge (2D vectors constructed with span- and pitch-wise velocity components)

Corsini *et al.* [35] derived a modified sinusoidal leading edge and applied it to a symmetric and a cambered NACA profile to study the dynamic of profile stall concluding that the sinusoidal leading edge decreased lift and increased drag at low angles of attack. At higher angles of attack, near stall and in post-stall conditions, the overall effect was to increase lift and mitigating the effects of stall, increasing the lift-recovery capability of the blade in post-stall conditions, Figure 13. This effect was correlated to the vorticity and vortical structures shed by the leading edge Figure 14, that at high angles of attack result in controlling the separation and limit it to the trailing edge portion corresponding to the troughs of the leading edge sinusoid, Figure 15.

In [36] Corsini *et al.* applied the same approach to an industrial fan blade, aiming at controlling the evolution of stall. The selected fan blade leading edge (JFM224) was modified in the upper 25% of the span with a sinusoidal shape (JWFM224). The profile was limited to this region as the fan was loaded only at the tip in this application, Figure 16.

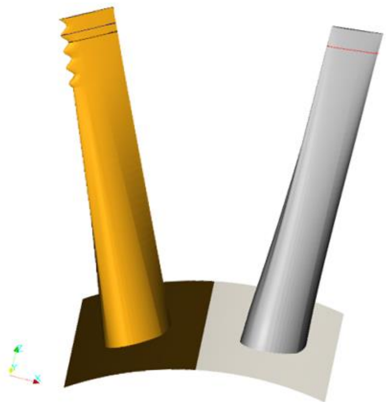

Figure 16. Modified (left) and datum (right) fan blade [36]

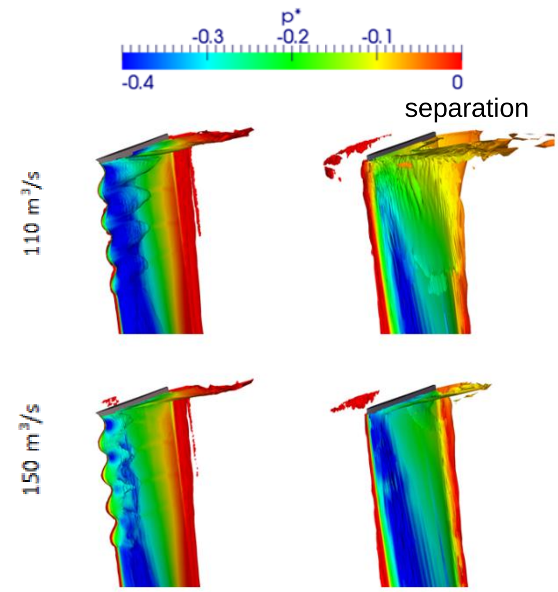

Figure 17. Turbulent structures highlighted with vorticity isosurfaces for JFM224 and JWFM224 at peak pressure (top) and maximum flow rate (top) [36]

The modified blade resulted having a decreased pressure rise capability in the stable range of operations, but a flatter pressure curve in stalled operations. Efficiency was the same at design point and increased at lower flow rates, with a slight drop only at the higher flow rates. This confirmed that the shaping could be applied to turbomachinery to affect the dynamic of stall. It also confirmed that the effect was related to the vorticity shed by the leading edge bumps, that modified the evolution of tip leakage vortex, reducing tip separation at lower mass flow rates, Figure 17. It was also found that the dynamics was not affected in the stable range of operations of the fan.

3.3. Aerofoil noise reduction with soft coating

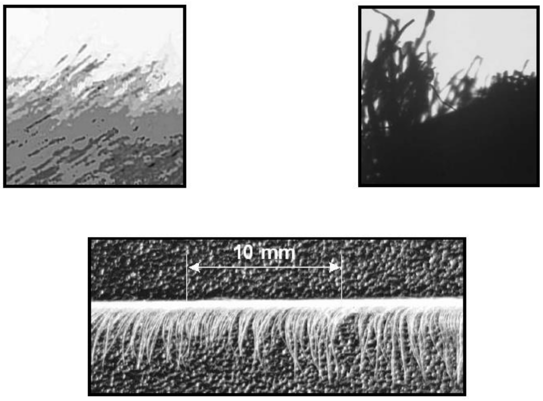

Figure 18. Photographs of the fuzzy surface of the owls leading edge (left), velvet (right) and owl feather with filaments (bottom) [37]

Following the evidence that the filaments on the owl wings, Figure 18, reduce the noise whilst flying and allow the bird to hunt being heard by preys [38], Vad *et al.* [37], applied a velvet surface coating to an airfoil to measure aerodynamic and aeroacoustic performance and explore the possibility of using this strategy to control fan noise. They chose velvet as it has a morphology similar to that of the owl filaments.

They found that the coating effectively reduced the sound pressure level, Table 1, and in particular in a frequency range critical from a human point of view, but also that it decreased lift and increased drag, Table 2.

Table 1. A-weighted Sound Pressure levels [37**]**

Test case	Incidence	L_A [dB(A)]
Uncoated	0 deg	64.4
Coated		63.0
Uncoated	5 deg	63.0
Coated		62.7
Uncoated	15 deg	74.7
Coated		73.8

Table 2. Lift and drag coefficients [37**]**

Test case	Incidence	C_L	C_D
Uncoated	5 deg	0.75	0.03
Coated		0.65	0.08
Uncoated	15 deg	1.45	0.12
Coated		0.85	0.61

4. GEOMETRY DONE BY MOTION, THE SCIENCE OF PATTERN

Leonardo was well aware of the critical role of mathematics in the formulation of scientific ideas and in the recording and evaluation of experiments. “There is no certainty,” he wrote in his Notebooks, “where one can not apply any of the mathematical sciences, nor those which are connected with the mathematical sciences.” But at the same time there was no mathematical language appropriate to express the kind of science he was addressing, i.e. the explorations of the forms of nature in their movements and transformations. For this reason, Leonardo exploited his visual approach with new techniques that foreshadowed branches of mathematics that would not be developed until centuries later. To mention but a few, the theory of functions and the fields of integral calculus and topology [1]. Instead of mathematics, he frequently used his exceptional drawing facility to document in graphic form his observations in pictures that are often strikingly beautiful while, at the same time, they take the place of mathematical diagrams. To some extent, Leonardo used symbolic languages and analyses. This is evident in his praise of geometry being “the prince of mathematics”.

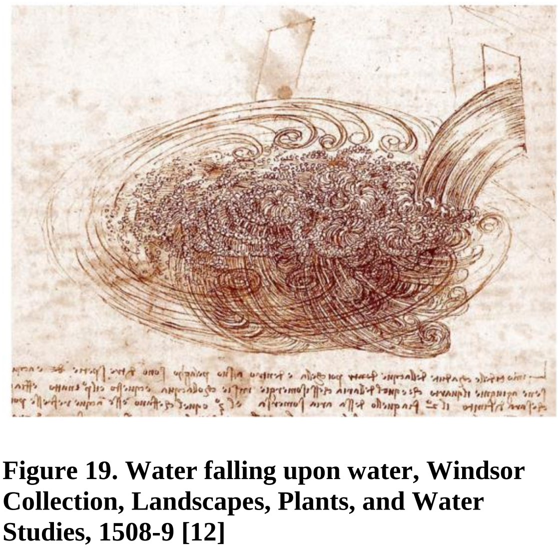

Figure 19. Water falling upon water, Windsor Collection, Landscapes, Plants, and Water Studies, 1508-9 [12]

His celebrated drawing of “Water falling upon water” (Figure 19), for example, is not (only) a realistic snapshot of a jet of water falling into a pond, but an elaborate visualization to educt several types of turbulent flow structures caused by the impact of the jet. Those drawings are diagrammatic representations of the functional relationships between various parts of the phenomenon under observation. According to Arasse [39], the drawings are the results of (repeated) observations being crystallized in the form of a synthetic model of the underlying patterns (e.g. in Figure 19 the vortex decay from largest to smallest scales). From a technical viewpoint, Leonardo relied on *sfumato* technique as a tool to give back the dynamics of the phenomenon.

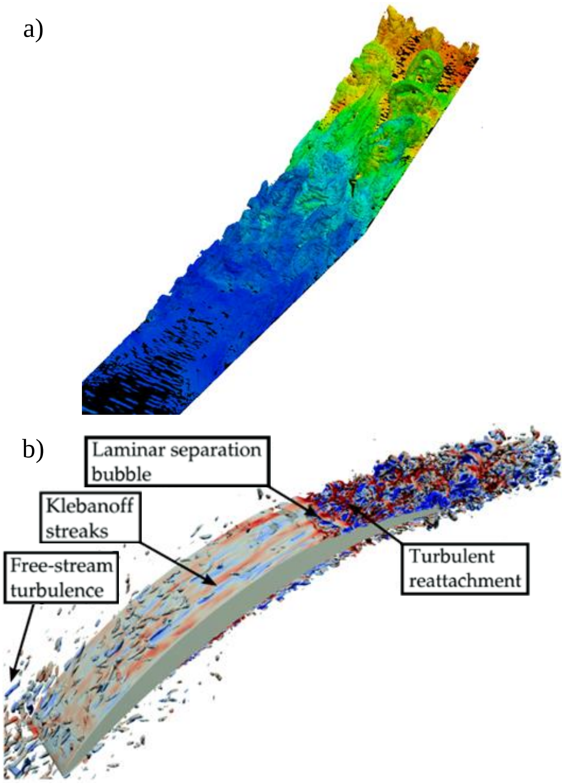

Figure 20. a) LES of shock-wave-boundary layer interaction [40], and b) boundary layer development over the suction surface of a linear compressor cascade [41]

A remarkable analogy to the dynamics of flow at the diverse scale given in “Water falling upon water”, is the one that results from high-fidelity simulations of turbulent flow under in complex geometries and adverse pressure gradients. Figure 20, respectively, shows the results of LES to compute the interaction between a shock-wave and a turbulent boundary layer (Figure 20.a), and the boundary layer development over the suction surface of a linear compressor cascade (Figure 20.b).

Geometry was the ideal “mathematical” test-bed in Leonardo’s view because of its potential to deal with continuous variables as it claimed in *Madrid Codices* “The mathematical sciences are only two, of which the first is arithmetic, the second is geometry. One encompasses the discontinuous quantities (i.e., variables), the other the continuous”. A theory for continuous quantities was needed to describe the unending transformations in nature (the dynamics of nature).

Specifically, Leonardo studied continuous transformations of rectilinear and curvilinear shapes to model movement and transformation as processes of continual transition, in which “everything that moves the space which it acquires is as great as that which it leaves” (*Madrid Codices*). He defined the principle of conservation of volume (continuity) as a general one governing all changes and transformations of natural forms (flow of water and other liquids). Here is how he writes about the flow of a river: “If the water does not increase, nor diminish, in a river, which may be of varying tortuosities, breadths and depths, the water will pass in equal quantities in equal times through every degree of the length of that river”. This definition was one of the examples of “geometry which is demonstrated with motion” (i.e. in Italian “*geometria che si prova col moto*”).

In a similar vein, Isogeometric analysis was recently developed as a computational approach able to integrate finite element analysis into conventional NURBS-based CAD design tools. Isogeometric analysis, first proposed in 2005 by Hughes and his co-workers [42], employs complex NURBS geometry in the FEA test-bed. Figure 21 illustrates the peculiarities of IGA analysis in turbo-charger space-times iso-geometric simulation [43].

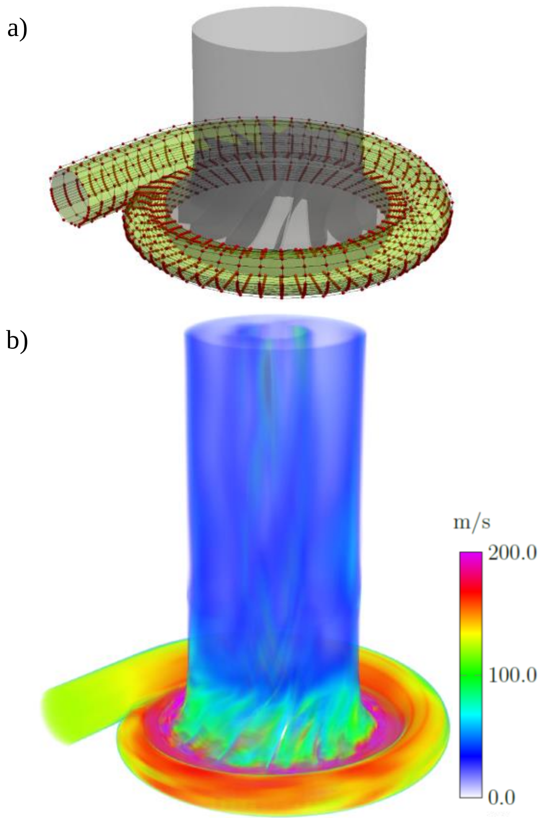

Figure 21. Turbine a) NURBS mesh and b) velocity field [43]

Looking at this geometry of continuous transformations, or mappings, from today's perspective (e.g. complexity theory), Leonardo can also be seen as the inventor of the branch of mathematics now known as topology [1]. As an example of Leonardo's attraction for topology, the frescos He realized in Milan depicted tangled labyrinths of knots, Figure 22.

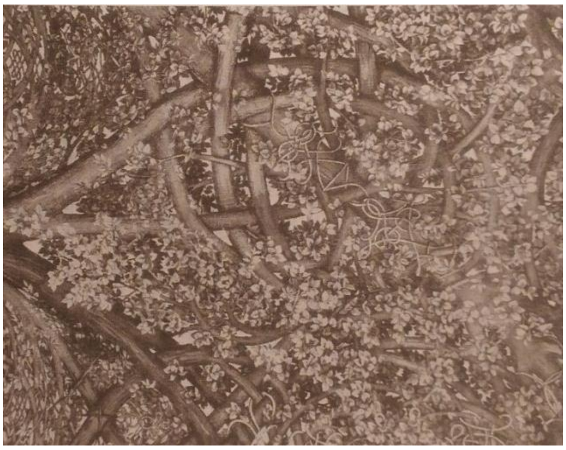

Figure 22. Sala dell'Asse, Castello Sforzesco, Milano, (1498-9)

Here the knots and scrolls, known as *fantasie dei vinci* in the fifteenth century, were depicted to refer to his surname Vinci whose semantic meaning stands for reeds used in basketry.

In this respect, during the last decades, a number of scholars have proposed and used the general theorems of topology as a methodology to classify tree-dimensional separations and, correspondingly, the limits in turbomachines operation [44, 45]. The original idea dated back to Poincaré [46] proposal on singular points of Navier-Stokes PDEs, and was later implemented by various authors trying to associate the behaviour of flow with pattern lines emanating from critical points [47-50]. Figure 23 shows the critical points classification given by Dallmann [51].

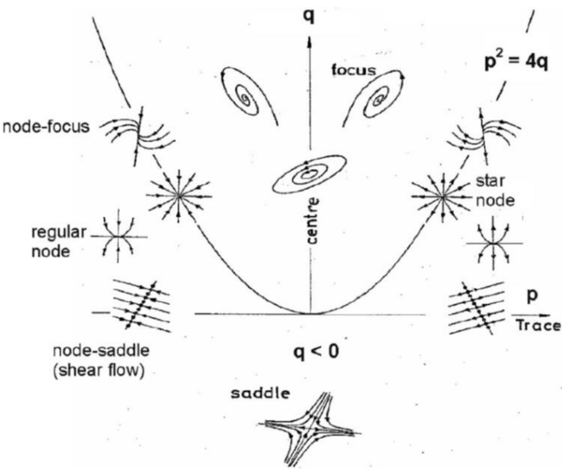

Figure 23. Classification of critical points [51]

Furthermore, Figure 24 shows examples of topology studies in axial compressor cascade (Figure 24.a) or in an annular stator passage (Figure 24.b).

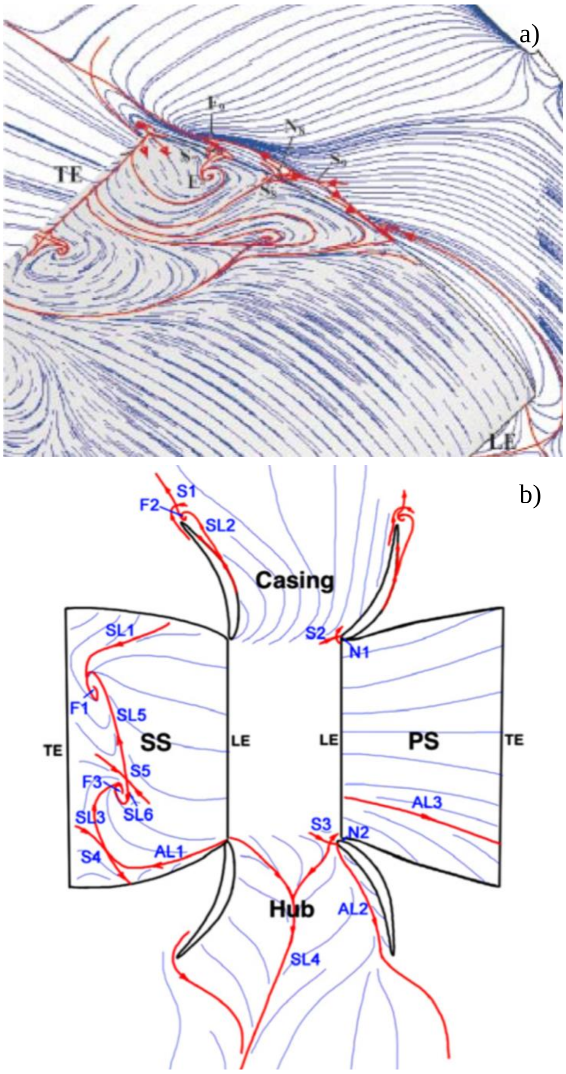

Figure 24. Examples of topology studies in turbomachines: a) axial compressor cascade [44], and b) annular stator passage [45].

An additional aspect of Leonardo's modernity is still interlinked to complexity theory and the new mathematical language developed for dynamics (or non-linear) complex system (including the turbulent flows and growth patterns of plants studied by Leonardo). Specifically, this new language is no longer represented by algebraic relationships, but geometric shapes, like computer-generated strange attractors analyzed in terms of topological concepts.

In the turbomachinery fluid dynamics, few studies have been published advocating the use of strange attractors (and their topology features) in the identification of aerodynamic stall in signals measured by near-field or far-field sensors.

The time-domain signal analysis was based on the phase-space portraits (or representations), following the stall detection through acoustic methods already investigated by Bianchi and co-workers [52-55] which proposed the use of symmetrised dot pattern representation of pressure sound signals in order to differentiate between critical and non-critical stall conditions, and to identify stall precursors.

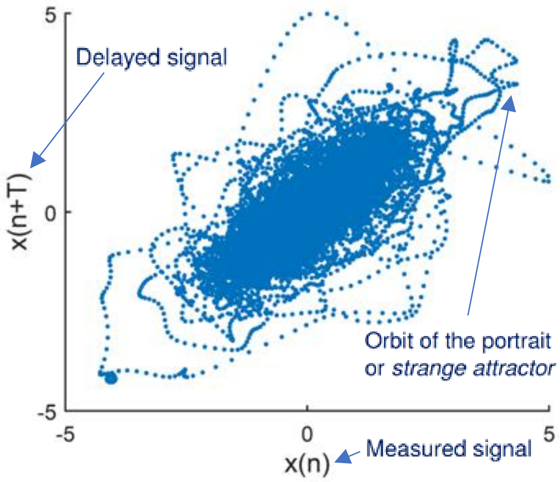

Figure 25. Phase-space portrait of time resolved pressure signals during the stall of an axial fan [56]

Also, Palomba *et al*. [57] studied the chaotic dynamics on which the rotating stall is based. They reconstructed the phase space portraits of velocity, static pressure and vibration experimental signals data series, using the delay method. Their idea was to represent the system dynamics and the transient phenomena using a non-linear tool, on the basis of patterns identification and trajectories inspection. With the phase space reconstruction it is possible to do the embedding of a univariate sequence of data (the signal considered as a time series) in a phase-space portrait evaluating the time lag T and the embedding dimension D, so to obtain D vectors from the original signals using T as the time delay. The pattern reconstruction makes use of "method of delays" first proposed by Takens [58]. Figure 25 illustrates the derivation of a portrait in the phase-space from time resolved pressure signals [56].

This mathematics, far more sophisticated than what Leonardo envisaged, corresponds to the Leonardo's intuition of measuring the world's complexity using the "geometry done with motion" metric [1].

5. CONCLUSIONS

The field of fluid dynamics (aka "fluid mechanics") is complicated due to the pervasive appearance of turbulent flows far from being confined into a unique comprehensive mathematical analysis. In an oft-quoted phrase, Richard Feynman (physicist and Nobel laureate) called turbulence "the last unsolved problem of classical physics". Turbulent flows are composed of eddies, also known as vortices, in a broad range of sizes, continually forming and breaking down swirling and randomly moving patches. Those patterns fascinated Leonardo and his observations, drawings and notes on the dynamics of water remained undiscovered for several centuries after his death, causing his irrelevance to the development of science and engineering.

Moving in time, the first theoretical analyses of fluids were undertaken only in the eighteenth century when i. the mathematician Leonhard Euler applied the Newtonian laws of motion to a "perfect" fluid, and ii. the physicist and mathematician Daniel Bernoulli discovered some of the basic energy relations in liquids. In the nineteenth century Claude-Louis Navier and George Stokes formulated the generalized Newton's equations for the description of the flow of viscous fluids. And, the mechanical engineer Osborne Reynolds discovered that the onset of turbulence can be characterized in terms of a single parameter (aka the Reynolds number), which is dependent upon flow velocity, fluid's viscosity, and length scale.

This is the turning point where physicists and mathematicians rediscovered many of the theoretical ideas about fluid motion that Leonardo had clearly formulated centuries earlier. The novelties he brought to fluids observations are consequence of his systematic viewpoint. Without a solid mathematical background, he was able to create a qualitative demonstration of nature, made of visualization, able to understand the dynamics of fluids through geometry and transformations.

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