Conference on Modelling Fluid Flow (CMFF'03)
The 12th International Conference on Fluid Flow Technologies
Budapest, Hungary, September 3 - 6, 2003

WATER HAMMER IN ELASTIC PIPELINES

Olimpia BLĂGOI, professor*

Department of Hydrotechnical Constructions and Sanitary Engineering
Technical University "Gh. Asachi" of Iasi

*Corresponding author: Olimpia Blãgoi, Bd. Prof. Mangeron, Nr. 63-65, 6600 Iassy, Romania
Tel.: (+40 0232) 278683, Fax: (+40 0232) 270804, Email: oblagoi@yahoo.com

ABSTRACT

This is a presentation of a general formula for the celerity of the waves in elastic pipes and an attempt to estimate a mathematical model of an unsteady flow in elastic pipelines after rapid valve closure. In order to solve the system of differential partial equations attached to this model, two numerical procedures are used: the method of characteristics and the Galerkin's criterion. The variation of the piezometric head in valve section is analyzed bellow.

Key Words: Galerkin's method, method of characteristics, shock waves

NOMENCLATURE

c	[m/s]	wave celerity
c_0	[m/s]	wave velocity in a liquid at rest (sound travel velocity in water having $t = 10°$ C)
g	[m/s^2]	acceleration of gravity
r	[m]	internal radius
r_1	[m]	internal radius of the excavation cross-sectional area
t	[s]	time
t_r	[s]	reflexive time
v	[m/s]	liquid velocity
v_0	[m/s]	liquid velocity in the stationary state
A_0	[m^2]	cross-section area
D	[m]	pipe diameter
E_l	[N/m^2]	elasticity modulus of the liquid
E_p	[N/m^2]	Young's modulus of the pipe material
E_c	[N/m^2]	Young's modulus of the concrete backfill
E_r	[N/m^2]	Young's modulus of the rock
E_{st}	[N/m^2]	Young's modulus of elasticity of the steel
H_0^{up}	[N]	initial upstream piezometric head
H_0^{ds}	[N]	initial downstream piezometric head
H_0^{v}	[N]	valve head in the initial moment
L	[m]	pipe length
Q_0	[m^3/s]	discharge in the stationary state
A	[-]	Coriolis coefficient
β	[-]	Boussinesq's coefficient
γ	[N/m^3]	specific weight of liquid
δ	[m]	shell thickness
λ	[-]	Darcy-Weisbach friction factor
μ	[-]	reinforcement factor
ρ	[kg/m^3]	liquid density
υ_r	[-]	Poisson's ratio of the rock
ψ	[-]	coefficient depending on the pipe elasticity

1. SHOCK WAVE PROPAGATION

In any hydraulic system, the pressure surges occur during the transition from one steady state to another. Depending on the time interval of the transients, pressure waves travelling at speeds close to that of the sound waves can easily generate enough energy to damage the system. This is the so-called water hammer effect that can cause so much damage to pipe, valves and other equipment.

The water hammer propagates along the pipeline like elastic wave with the celerity c that depends on the pipes elasticity, fluid compressibility and support conditions.

The overpressure produced by this transient phenomenon is very important for the engineering design of the pressure pipe as well as the suction to avoid the pipe yield to axial compression. The shock pressure value – positive or negative – depends on the closure time of the valve.

The speed of the shock wave along a pipe of geometrical and mechanical constant characteristics is given by the generalized formula [2,6]:

$$c = \frac{v}{2} + \frac{c_0}{\sqrt{\beta}\sqrt{1+\frac{E_l}{E_p}\psi}}\sqrt{1+\psi\frac{p}{E_p}} \tag{1}$$

with:

$$c_0 = \sqrt{E_l/\rho} \tag{2}$$

For simple pipes, the coefficient ψ depends on the material of the walls only, so:

– for steel:

$$\psi = \frac{D}{\delta} \tag{3}$$

– for concrete:

$$\psi = \frac{1}{1+9.5\,\mu}\cdot\frac{D}{\delta} \tag{4}$$

For complex pipes with variable thickness and diameter, the equivalent value of ψ is:

$$\psi = \frac{D}{\delta} = \frac{\sum_i \frac{L_i\, D_i}{\delta_i}}{\sum_i L_i} \tag{5}$$

For thick – walled pipes, the celerity of the pressure wave is computed with the formula:

$$c = \frac{v}{2} + \frac{c_0}{\sqrt{\beta}\cdot\sqrt{1+2\frac{E_l}{E_p}\frac{(r+\delta)^2+r^2}{(r+\delta)^2-r^2}}} \tag{6}$$

For steel lined shafts the used formula is:

$$c = \frac{v}{2} + \frac{c_0}{\sqrt{\beta}\cdot\sqrt{1+2\frac{E_l}{E_p}\frac{r}{\delta}(1-\varphi)}} \tag{7}$$

where:

$$\varphi = \frac{\frac{r}{\delta}}{\frac{r}{\delta}+\frac{E_{st}}{E_c}\frac{r_1^2-r^2}{2\,r\,r_1}+\frac{E_{st}}{E_r}\frac{1+\upsilon_r}{\upsilon_r}} \tag{8}$$

The reflexive time or the phase is the time when the wave passes through the length of pipe and returns to the starting point:

$$t_r = \frac{2L}{c} \tag{9}$$

2. ANALYTICAL CONSIDERATIONS

In the development of the mathematical model, there is an unsteady state when the pipeline is firmly clamped.

The following assumptions concerning the pipeline characteristics are taken now:

– geometrical: the cross-section area is constant; there is a valve downstream, whose movement is defined by the law $A = A(t)$ until the total obtrusion of the useful section;

– hydraulically: upstream acts a constant pressure H_0^{up} but downstream acts the pressure H_0^{ds}.

The analysis of [6,8] is used for the continuity equation in unsteady flow:

$$\frac{\partial(\rho\, Q)}{\partial s} + \frac{\partial(\rho\, A)}{\partial t} = 0 \tag{10}$$

with:

$Q = Q(t,s)$; $\rho = \rho[p(t,s)]$; $A = A[p(t,s)]$

and generalized formulation:

$$A\left(1+\frac{E_l}{E_p}\frac{D}{\delta}\right)\frac{\partial H}{\partial t} + Q\,\frac{\partial H}{\partial s} + \frac{c_0^2}{g}\frac{\partial Q}{\partial s} + Q\, I_0 = 0 \tag{11}$$

The same for the dynamic equation of the wave in unsteady flow:

$$\frac{\partial H}{\partial s} + \frac{\beta}{g}\frac{\partial v}{\partial t} + \frac{\alpha}{2g}\frac{\partial v^2}{\partial s} + J = 0 \tag{12}$$

where $J = S_0\, Q\, |Q|$.

Finally:

$$\left(1-\alpha\frac{\rho}{E_p}\frac{D}{\delta}\frac{Q^2}{A^2}\right)\frac{\partial H}{\partial s} - \beta\frac{\rho}{E_p}\frac{D}{\delta}\frac{Q}{A}\frac{\partial H}{\partial t} + \frac{\beta}{g\,A}\frac{\partial Q}{\partial t} + \frac{\alpha}{g}\frac{Q}{A^2}\frac{\partial Q}{\partial s} + J - \alpha\frac{\rho}{E_p}\frac{D}{\delta}\frac{Q^2}{A^2}I_0 = 0 \tag{13}$$

The generalized formulation of [6] and also the assumptions that:

$\gamma H / E_p << 1$; $v_0 << c$; $\alpha \cong \beta \cong 1$

are taken into account now.

A system of differential equations with partial derivatives, quasi-linear, of hyperbolic type to describe the phenomenon is obtained, so:

$$\begin{cases} \left(1+\dfrac{E_l}{E_p}\dfrac{D}{\delta}\right)\dfrac{\partial H}{\partial t}+\dfrac{Q}{A}\dfrac{\partial H}{\partial s}+\dfrac{c^2}{g\,A}\dfrac{\partial Q}{\partial s}+\dfrac{Q}{A}I_0=0 \\ -\dfrac{\rho}{E_p}\dfrac{D}{\delta}\dfrac{Q}{A}\dfrac{\partial H}{\partial t}+\left(1-\dfrac{\rho}{E_p}\dfrac{D}{\delta}\dfrac{Q^2}{A^2}\right)\dfrac{\partial H}{\partial s}+ \\ +\dfrac{1}{gA}\dfrac{\partial Q}{\partial t}+\dfrac{Q}{gA^2}\dfrac{\partial Q}{\partial s}+J-\dfrac{\rho}{E_p}\dfrac{D}{\delta}\dfrac{Q^2}{A^2}I_0=0 \end{cases} \tag{14}$$

The attached matrix to the system (14) has the following form:

$$\begin{bmatrix} 1+\dfrac{E_l}{E_p}\dfrac{D}{\delta} & \dfrac{Q}{A} & 0 & \dfrac{c^2}{g\,A} & \dfrac{Q}{A}I_0 \\ -\dfrac{\rho}{E_p}\dfrac{D}{\delta}\dfrac{Q}{A} & 1-\dfrac{\rho}{E_p}\dfrac{D}{\delta}\dfrac{Q^2}{A^2} & \dfrac{1}{g\,A} & \dfrac{Q}{g\,A^2} & J-\dfrac{\rho}{E_p}\dfrac{D}{\delta}\dfrac{Q^2}{A^2}I_0 \end{bmatrix} \tag{15}$$

3. NUMERICAL SOLUTION

3.1. Characteristics Method

The solution of the hyperbolic partial equations using the method of characteristics is presented in what follows. Essentially, the adopted procedure is to decompose the partial differential equations into a system of total differential equations. Thus, the system (14) becomes:

$$\begin{cases} ds = a\,dt \\ dH + p\,dQ + r\,dt = 0 \\ ds = b\,dt \\ dH + p'\,dQ + r'\,dt = 0 \end{cases} \tag{16}$$

where a, b, p, p', r, r' are the determinants:

$$a\,;b=\frac{(1{,}4)+(2{,}3)\pm\sqrt{[(1{,}4)+(2{,}3)]^2-4\,(1{,}3)\,(2{,}4)}}{2\,(1{,}3)}$$

$$p=\frac{(1{,}3)\,a-(2{,}3)}{(1{,}2)};\quad r=\frac{(1{,}5)\,a-(2{,}5)}{(1{,}2)}; \tag{17}$$

$$p'=\frac{(1{,}3)\,b-(2{,}3)}{(1{,}2)};\quad r'=\frac{(1{,}5)\,b-(2{,}5)}{(1{,}2)}$$

The determinants (17) having the elements of i and j columns of the matrix (15) are denoted by the symbol (i, j).

The system with a single water-piping tract (**Figure 1**) with a local hydraulic resistance (a valve) is analyzed now.

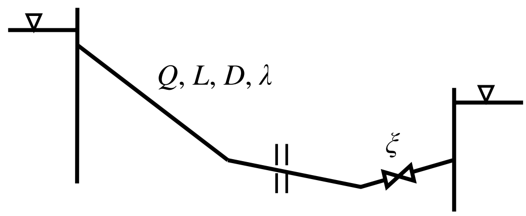

Figure 1. Hydraulic system scheme

To solution the system it must add the initial and boundary conditions. In this way, the following valve operation law is adopted (**Figure 2**):

$$A(t)=\begin{cases} A_0-\dfrac{A_0-A_1}{T_1}\,t \quad for\ t\in[0,T_1] \\ \dfrac{A_1}{T_2-T_1}\,(T_2-t)\ for\ t\in[T_1,T_2] \end{cases} \tag{18}$$

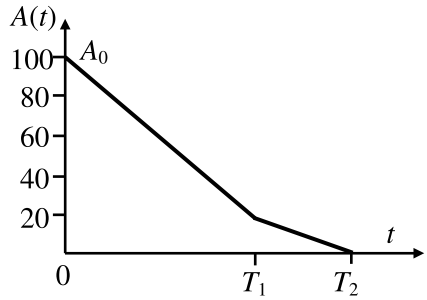

Figure 2. Valve operational law

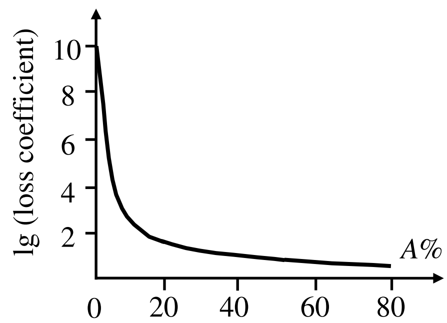

Figure 3. Hydraulic characteristic of the valve

Also the flow system characteristics are:

- loss coefficient of the valve (**Figure 3**) at total closure (16.61) for direct sense and (18.66) for the opposite;

- characteristic piezometric heads;
- pipe length 1,600 m, pipe diameter $D = 300$ mm;
- $\lambda = 0.018$;
- discharge $Q_0 = 0.0461$ m^3/s, piezometric heads from a study level $H_0^{up} = 40$ m, $H_0^{ds} = 30$ m, $H_0^v = 38.82$ m in the initial moment.

The results obtained for the evolution of the piezometric head upstream in proximity to the valve are shown in **Figure 4**.

3.2. Galerkin's Variational Method

The use of this method allows a simple formulation, but it is noticed that it leads to a large extent of the iterative procedure.

The variables are the discharge and the piezometric head. A combination of the algebraic functions for these variables is given. Assuming that the spatial axis „*s*“ is the *x* ones, the combination chosen may be the following:

$$Q(x,t) = \sum_{i=1}^{n} u(t) N_i(x); \tag{19.a}$$

$$H(x,t) = \sum_{i=1}^{n} w(t) N_i(x) \tag{19.b}$$

where $N_i(x)$ are chosen as interpolating functions, but $u(t)$ and $w(t)$ are the functions of time to be determined.

Thus, the system (14) becomes:

$$\int_0^L \{syst.(14)\} \cdot N_i(x)\, dx = 0, \ (i = 1, \ldots, n) \tag{20}$$

Substituting Eqs. (19) into Eqs. (20), these can be rewritten as:

$$\begin{cases} \left(1 + \dfrac{E_I}{E_p}\dfrac{D}{\delta}\right)[A_{ij}]\left[\dfrac{dw}{dt}\right] + [B_{ij}][w] + [E_{ij}][u] + [F_i] = 0 \\ [C_{ij}]\left[\dfrac{dw}{dt}\right] + [G_{ij}][w] + \dfrac{1}{gA}[A_{ij}]\left[\dfrac{du}{dt}\right] + \\ + \dfrac{1}{gA}[B_{ij}][u] + [D_i] = 0 \end{cases} \tag{21}$$

where:

$$A_{ij} = \int_0^L N_j N_i\, dx; \quad B_{ij} = \int_0^L \frac{Q}{A}\frac{dN_j}{dx} N_i\, dx;$$

$$C_{ij} = -\frac{\rho}{E_p}\frac{D}{\delta}\int_0^L \frac{Q}{A} N_j N_i\, dx;$$

$$D_i = \int_0^L \left(J - \frac{\rho}{E_p}\frac{D}{\delta}\frac{Q^2}{A^2} I_o\right) N_i\, dx; \tag{22}$$

$$E_{ij} = \frac{c^2}{g \cdot A}\int_0^L \frac{dN_j}{dx} N_i\, dx; \quad F_i = \int_0^L \frac{Q}{A} I_0\, N_i\, dx;$$

$$G_{ij} = \int_0^L \left(1 - \frac{\rho}{E_p}\frac{D}{\delta}\frac{Q^2}{A^2}\right)\frac{dN_j}{dx} N_i\, dx$$

where $i, j = 1, \ldots, n$

The initial conditions can be expressed now by:

$$\int_0^L Q(x,0) N_i(x)\, dx = \int_0^L Q_0(x)\, N_i(x)\, dx;$$

$$\int_0^L H(x,0)\, N_i(x)\, dx = \int_0^L H_0(x)\, N_i(x)\, dx \tag{23}$$

It is noticeable that, as compared to the Method of Characteristics, the boundary conditions seem more difficult to be introduced, because the complete solution must be at each step evaluated. The subsystem cannot be independently solved.

The $N_i(x)$ functions take the polynomial form of third order:

$$N_1 = 1 - \frac{3x^2}{L^2} + 2\frac{x^3}{L^3}; \quad N_2 = \frac{x}{L} - 2\frac{x^2}{L^2} + \frac{x^3}{L^3};$$

$$N_3 = \frac{3x^2}{L^2} - 2\frac{x^3}{L^3}; \qquad N_4 = -\frac{x^2}{L^2} + \frac{x^3}{L^3} \tag{24}$$

The calculated results allowed analyzing the variation of the piezometric head upstream the valve during the shock wave.

The obtained results are plotted in Fig. 4.

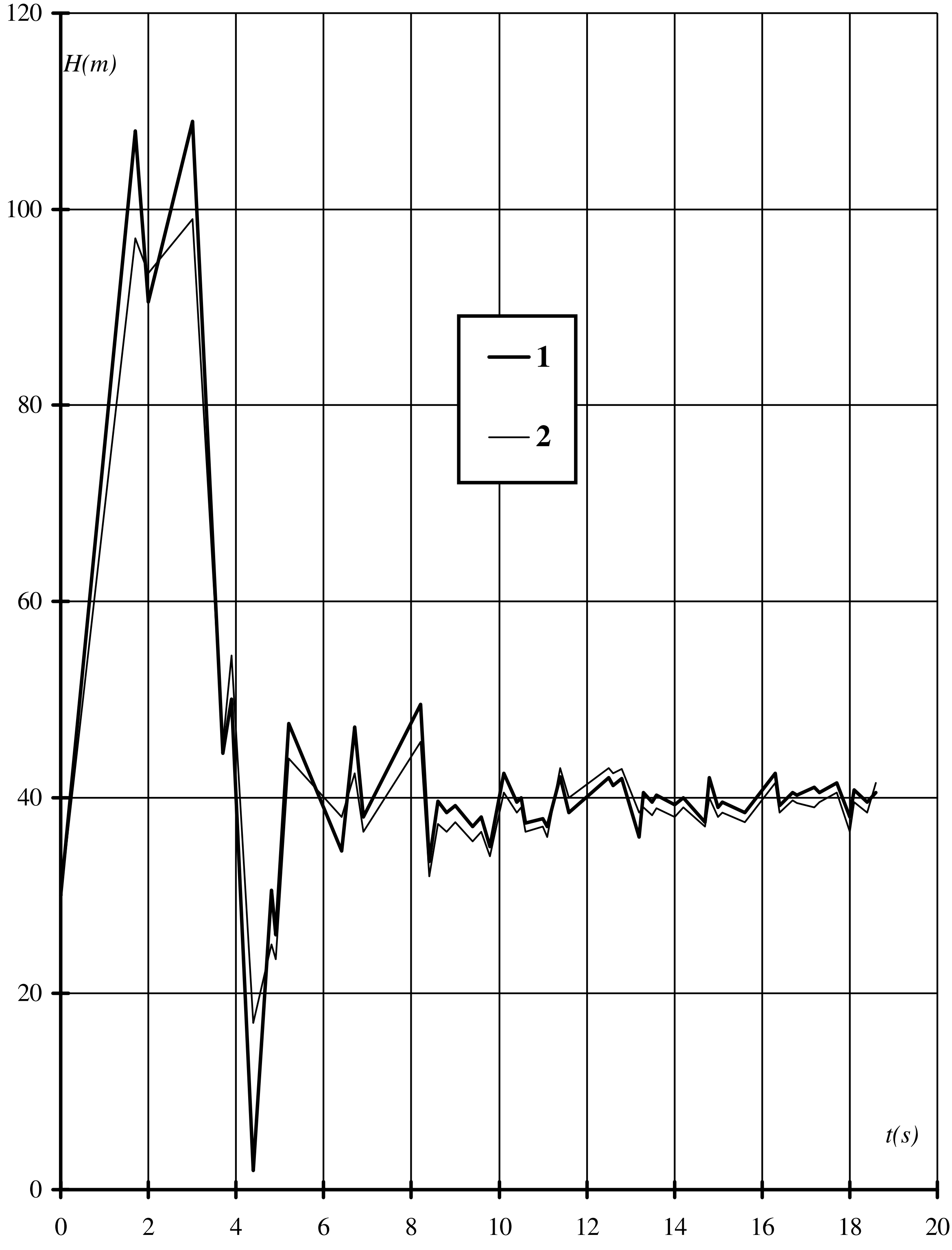

Figure 4 Pressure variation upstream the valve

1. Characteristics Method; 2. Galerkin's Method

4. CONCLUSIONS

The two used methods offer a good agreement and enough accuracy.

The Method of Characteristics may need a longer analyze interval with respect to the network of pipes, but it is easier.

The Galerkin's Criterion requires more computation times when dealing with a simple pipeline and can become very involved when the boundary conditions are applied.

Galerkin's Method may simply be generalized for the pipe network calculation.

It is necessary to introduce a certain degree of safety when designing the hydraulic system with rapid variable flow, since some adverse effects are occasionally neglected (e.g. transversal vibrations of the pipe wall, non-linear orifice law, minor irregularities in the process of closure, impact effects).

REFERENCES

[1] Acheson, D. J., 1990, Elementary Fluid, Clarendon Press, Oxford.

[2] Blăgoi, O., 1997, Bazele Hidraulicii, Editura Noël, Jassy.

[3] Dragoş, L., 1983, Principiile Mecanicii Mediilor Continue, Editura Tehnica, Bucureşti.

[4] Fayet, G., 1991, Hydraulique, Editions Eyrolles, Paris.

[5] Pezzinga, G., Scandura, P., 1995, "Unsteady Flow in Installations with Polymeric Additional Pipe", Journal of Hydraulic Engineering, Vol. 121, No. 11, p.802-812.

[6] Popescu, S., Alexandrescu, O., and Leu D., 1990, "Contribution à l'établissement du modèle mathématique du mouvement non-permanent dans les conduites à grande élasticité", Buletinul Institutului Politehnic din Iasi, Tom XXXVI (XL), fasc. 1-4, p. 31-38.

[7] Vardy, A.E., Hwang, K., 1991, "A Characteristics Model of Transient Friction", Journal of Hydraulic Research, 29 (5), p. 669-684.

[8] Walter, F.S.A., Chaudhry, H.M., 1997, "Computation of Energy Dissipation in Transient Flow", Journal of Hydraulic Engineering, Vol. 123, No. 2, p.108-116.

[9] Watt, C. S., Hobbs, J. M., and Boldy, A. P., 1980, "Hydraulic Transients Following Valve Closure", Journal of the Hydraulics Division, Vol. 106, p. 1627-1641.

[10] Wylie, E.B., Streeter, V., 1993, Fluid Transients in Systems, Prentices-Hall Inc., Englewood Cliffs, N.J.